EPFL

SCALING in MECHANICAL MICRO-SYSTEMS

Herbert Shea

Micro-470

v. 2024

Mechanical Scaling in MEMS

1. Intro: Scaling in Animals
2. Cantilevers: modes, stress, stress gradient
3. Non-linearities in MEMS and the Duffing equation
4. Thermo-mechanical Noise
5. Surface forces and Friction

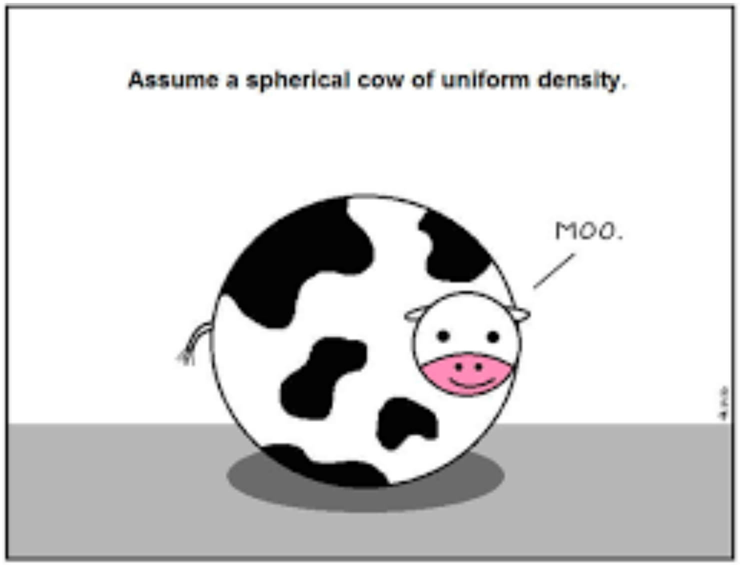
Simple scaling: parameter *L*

- One single parameter to describe all dimensions: ***L***

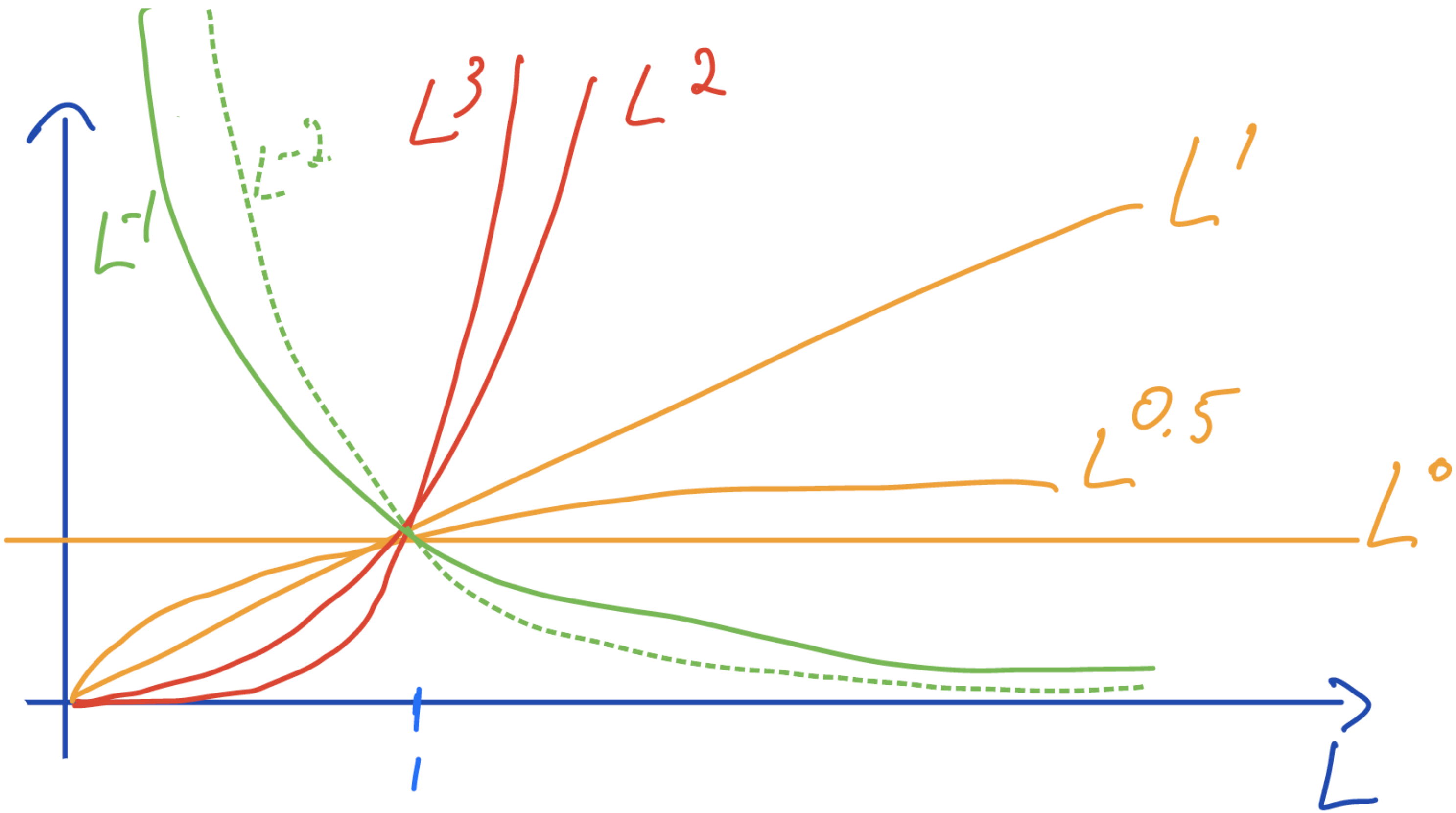
- Goal is overall rough scaling laws: what physical principles dominate at different length scale

- eg mass: $M \propto L^3$

  surface: $A \propto L^2$



EPFL



Simplified overview of mechanical scaling laws. 1 of 3

Parameter	Scaling Law	comment
Mass	$M \propto L^3$	
Area	$A \propto L^2$	
Surface-to-volume ratio	$\gamma \propto L^{-1}$	=> good for chemical reactions
Inertial forces	$F_{inertial} \propto M \propto L^3$	
Contact forces	$F_{contact} \propto A \propto L^2$	
Contact/inertial forces ratio	$\propto L^{-1}$	=> bad for manipulation
Van der Waals forces	$F_{VdW} \propto d^{-7}$	very short range!

Simplified overview of mechanical scaling laws: 2 of 3

Parameter	Scaling Law	comment
Spring Constant	$k \propto L$	calculated using Hook's law on a bar of cross-section A and length l: $k = \frac{AE}{l}$ (E: Young's modulus)
=> Spring (restoring) forces	$F \propto kx \propto L^2$	
Acceleration (intrinsic)	$a = \frac{F}{M} \propto L^{-1}$	
Natural frequency	$\omega_0 = \sqrt{\frac{k}{M}} \propto L^{-1}$	
"switching" time	$t_s = 1/\omega_0 \propto L$	
Viscous drag forces	$F_{vd} \propto \eta L v \propto L$	
Quality factor (=energy stored/energy loss per cycle)	$Q_f \propto \frac{kx^2}{F_f x_0} \propto L$	

Simplified overview of <u>mechanical</u> scaling laws: 3 of 3: energy

Parameter	Scaling Law	comment
Kinetic Energy	$E_{kin} = \frac{mv^2}{2} \propto L^3$	assuming v is constant. If $v \propto L$, then $E_{kin} \propto L^5$
Potential energy of a spring	$E_{pot,spring} = \frac{1}{2}kx^2 \propto L^3$	
Mechanical power	$P_{mec} = Fv \propto L^2$	
Mechanical energy density	$\frac{E_{pot,spring}}{m} = \frac{1}{2}\frac{\sigma^2}{E} \propto L^0$	scale invariant

In movies: isometric scaling

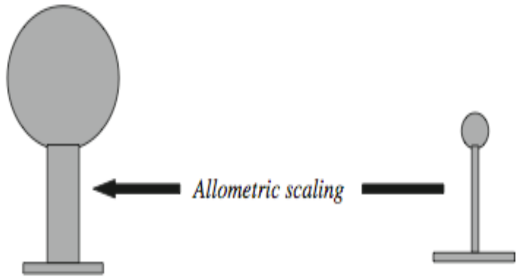
antman

1. In nature, allometric Scaling

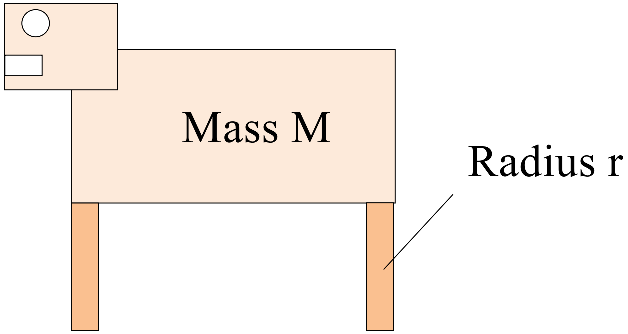
40 cm diameter

40 µm diameter

Aspect ratio changes when scale animals



Scaling of animal bone diameter

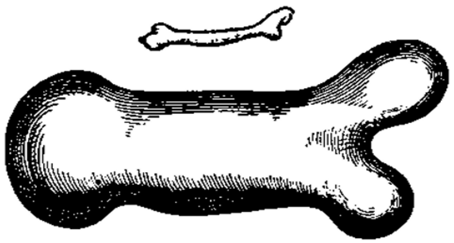


Assumptions:

- All animals have same bone material that fracture at a critical stress σ_{cr}
- Leg bone radius *r* is as small as possible while not fracturing from own weight

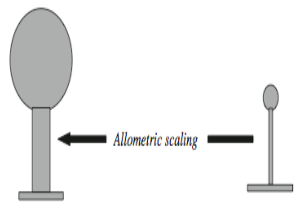
$$\sigma_{cr}\pi r^2 \backsim mg$$

$$r \propto \sqrt{m} \propto L^{3/2}$$



Galileo's depiction of the bones of light and heavy animals. (From *Dialogue on Two New Sciences*.) 1638

- This type of allometric scaling is a widely seen when miniaturizing mechanical systems

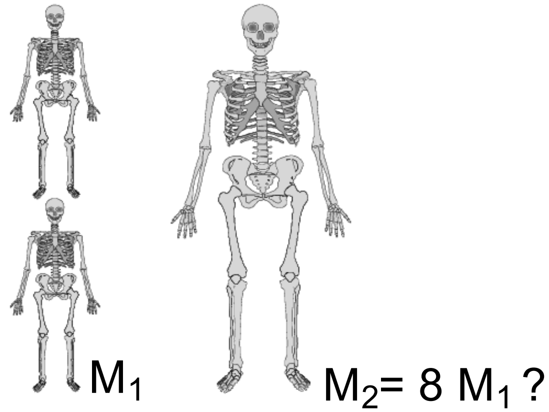


As it gets bigger, the animal eventually is only bone…

Mass of animal $\propto L^3$

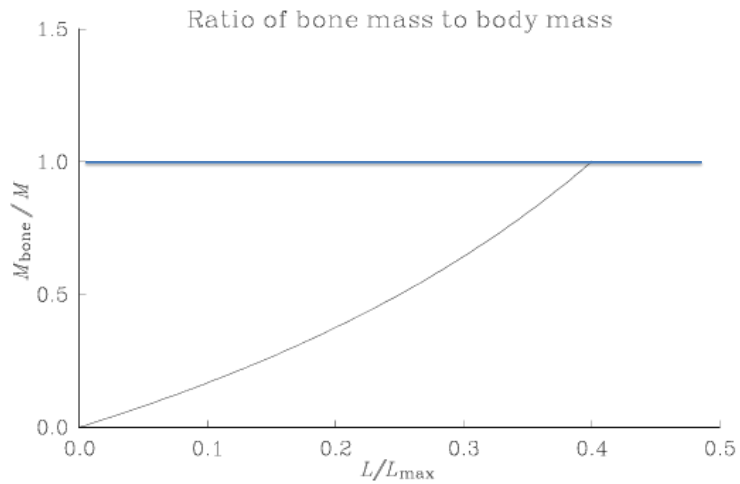
But strength of bones proportional to their cross-sectional area $\propto L^2$

The max force a muscle can exert in tension $\propto L^2$.



Strength to weight ratio (of proportionally scaled) animals scales as $\mathbf{L^{-1}}$

$$\frac{m_{bone}}{m_{tot}} = \frac{3}{2}\frac{L}{L_{max}}\frac{1}{1 - L_{max}}$$



Therefore larger animals change their proportions
– **larger bone and muscle diameter**

Max 60 tons, dinosaur

[J. V Noble, Physics of the human body, Virginia tech course](https://wiki.epfl.ch/scaling/documents/Papers/scaling_animals.pdf)
https://wiki.epfl.ch/scaling/documents/Papers/scaling_animals.pdf

Scaling of tree trunk diameter vs. tree height

Scaling Relationships

Allometric relationship: Height vs. diameter in trees

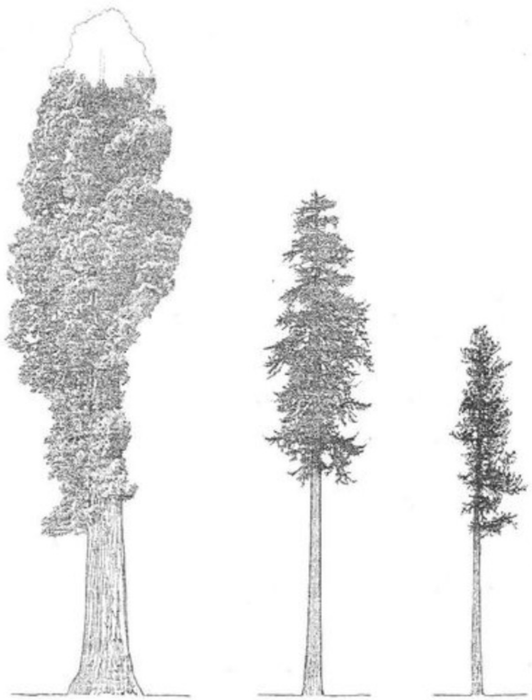
The critical buckling height for cylinders is:

$$H_{critical} = k * (E/\rho)^{1/3} * D^{2/3}$$

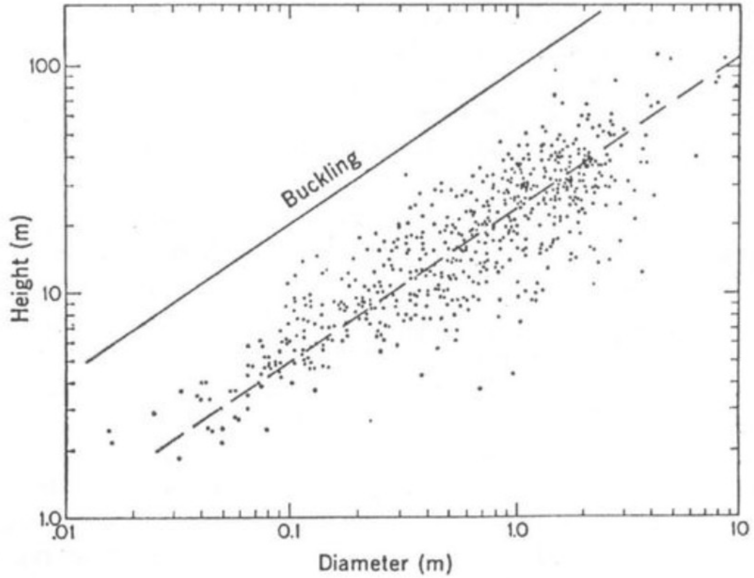
Therefore, if trees maintain "**elastic similarity**":

$$H \propto D^{2/3}$$

$$D \propto H^{3/2}$$



Dataset for U.S. record trees.



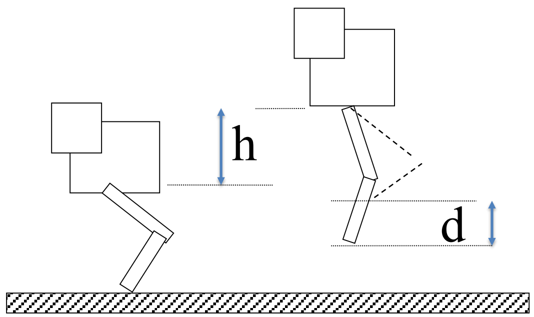
Both lines have slopes = 2/3; the broken line is 1/4 the magnitude of the complete line

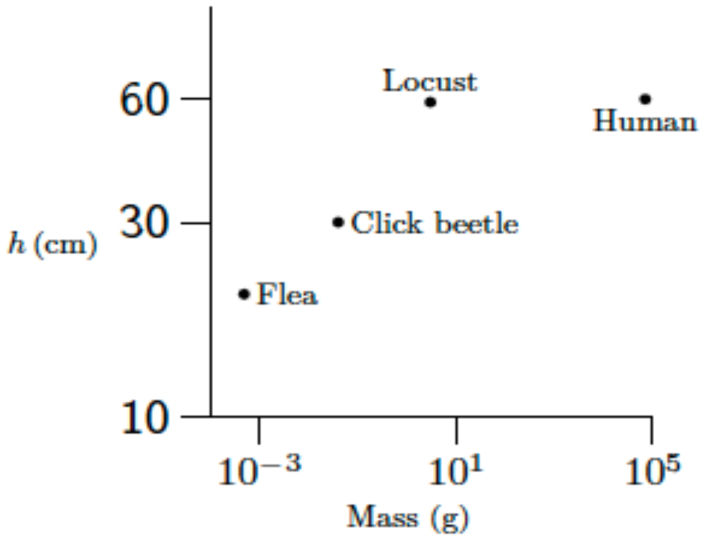
Trees avoid buckling under their own weight, with a 4x safety factor

See Greenhill (1881); Figure from McMahon (1975)

https://slideplayer.com/slide/12278649/

Scaling in nature: when size does not matter





from "Scaling: Why Animal Size is So Important", Knut Schmidt-Nielsen, Cambridge University Press; 1984

- Compute max Jumping height h
- Assume bone critical stress σ_{cr}
- d: elongation T: force from muscles

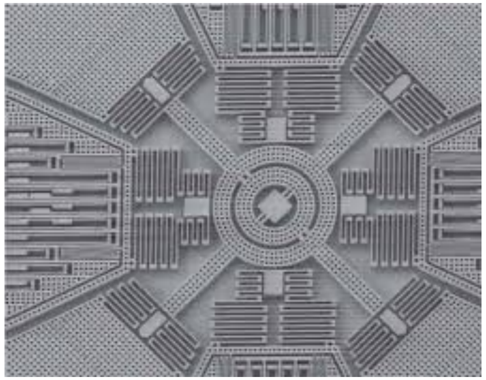
$$T_{max} = \sigma_{max} \cdot r^2 \qquad T_{max} d = Mgh$$

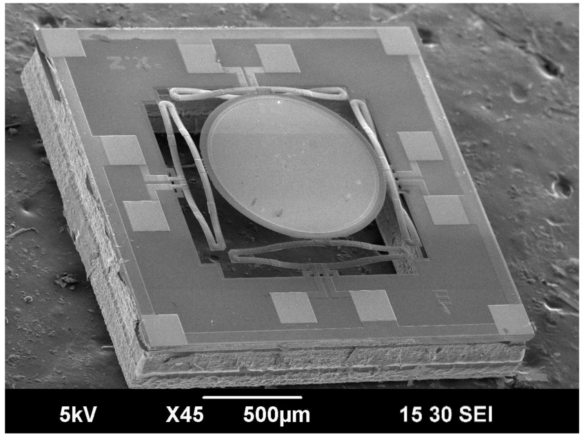
- If $r \propto L$ then $h = \frac{T_{max} d}{Mg} \propto \frac{L^2 L}{L^3} \propto L^0$

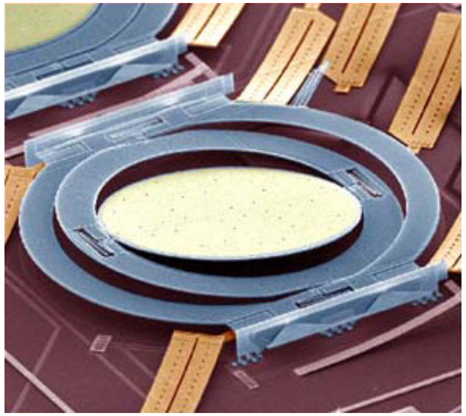
Max jumping height is independent of size !!

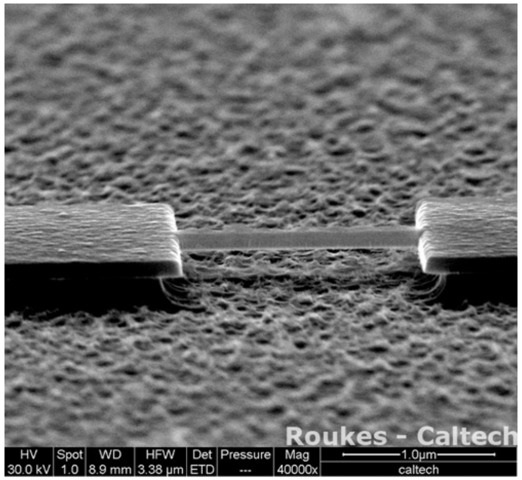
- If $r \propto L^{1.5}$ then $h = \frac{T_{max} d}{Mg} \propto \frac{L^2 L^{1.5}}{L^3} \propto L^{0.5}$

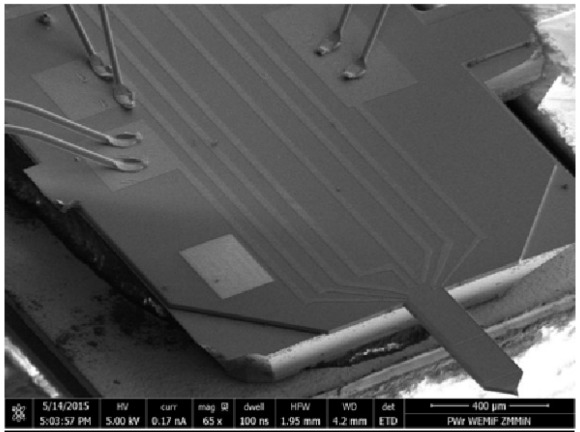
2. Cantilevers = Essential spring building block of MEMS

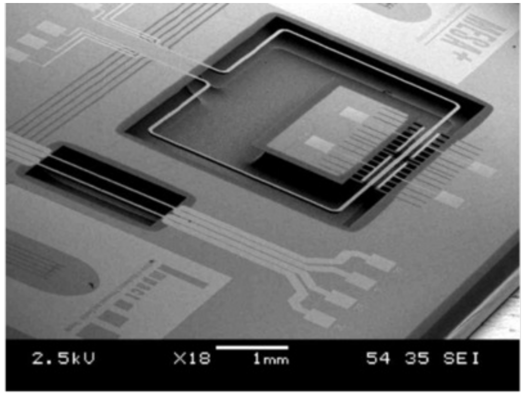












Cantilever: stress, strain and gravity

- Surface stress (at clamping edge) $$\sigma = \frac{6l}{wh^2} \cdot F$$

- Maximal allowed force F_{lim} at tip before fracture $$F_{\lim} = \sigma_{\max} \frac{wh^2}{6l} \quad \propto L^2$$

  where σ_{max} is fracture strain

Length l, width w, thickness h

- Deformation z_{lim} under F_{lim} $$z_{lim} = \frac{F_{max}}{k} = \frac{2\,\sigma_{max}}{3E}\frac{l^2}{h} \sim L$$

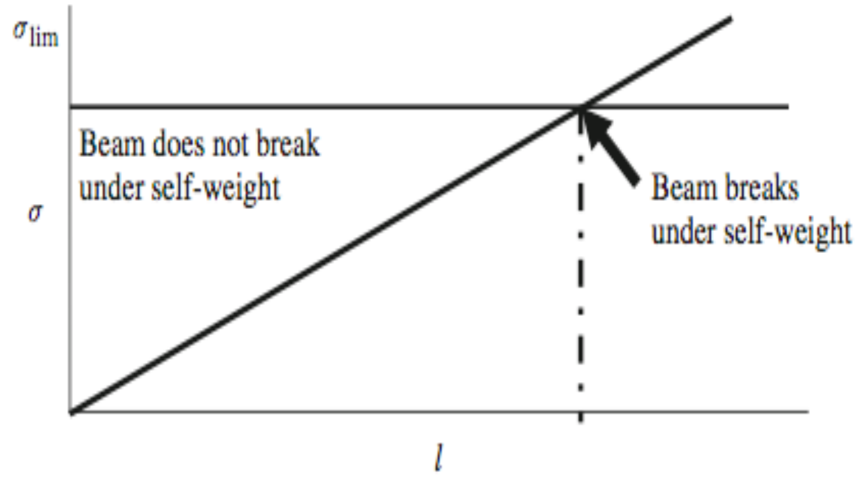
- Stress due to own weight (gravity) $$\sigma_x = \frac{\rho g l^2}{h} \quad \propto L$$

Cantilever: stress, strain and gravity

- Beam bending due to gravity $$\Delta z_g = \frac{3}{2}\frac{\rho g}{E}\frac{l^4}{h^2} \propto L^2$$
- Gravity bending can be related to resonance frequency

$$\Delta z_g \cong \frac{0.38}{f_0^{\,2}}$$

- <u>Numerical example</u>:
  - Poly-Si cantilever 2x2 µm^2, l=1 mm
  - => Δz_g=50 nm σ_x=0.01 MPa
  - Stress for 1 µm deflection σ_x =72 MPa (compare to yield stress of polysilicon: 2 GPa)
  - Considering a yield stress of 2 GPa, the beam would survive 200'000 G shock! (not true, as other factors are overlooked)



R1: Mechanics Over Micro and Nano Scales, S Chakraborty, Springer 2011, p.72

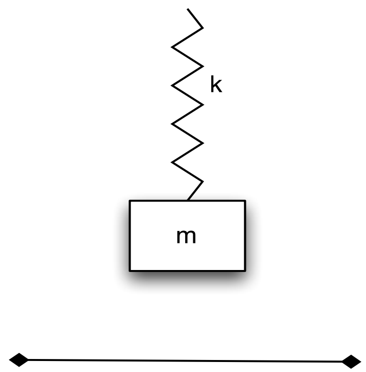
Accelerometer sensitivity (mass on a spring)

- To find the sensitivity, all we need to know is the resonance frequency !

$$S_x = \frac{x}{a} = \frac{m}{k} \propto L^2$$

- The mechanical sensitivity S_x to acceleration (x/a) can be fully determined by the resonance frequency, independently of mechanical parameters:

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad S_x = \frac{1}{\omega_0^{\ 2}}$$



- Example: ADLX50 ω_0=24 kHz => according to simple formula S_x=1.7 nm/g (this also means that the sensing element of accelerometer must be very sensitive to displacement).
  - Measured mechanical sensitivity in ADXL50 d*x*/d*a* = 0.43 nm/g

- Bandwidth vs. Sensitivity. Smaller minimum acceleration requires bigger S_x (for fixed x_{min}), hence a slower device.

http://www.iris.edu/hq/instrumentation_meeting/files/pdfs/MEMS_Seismology.pdf

Cantilever dynamics (or why size matters)

- First mode of cantilever: $f_1 = \frac{1}{2\pi}\omega_1 = \frac{1.03}{2\pi}\frac{h}{l^2}\sqrt{\frac{E}{\rho}} \qquad \propto L^{-1}$

- Numerical example: polysilicon cantilever (E=160 GPa) cantilever 2x2 μm^2
  - f_0=245 kHz for l=100 μm (first mode)
  - f_0=2.45 kHz for l= 1000 μm

Table 1: Fundamental Frequency *vs.* Geometry for *SiC*, [Si], and (GaAs) Mechanical Resonators

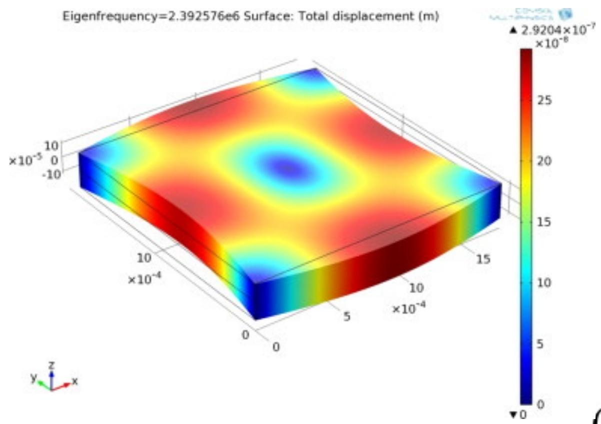
Boundary Conditions	Resonator Dimensions ($L \times w \times t$, *in* μm)			
	$100 \times 3 \times 0.1$	$10 \times 0.2 \times 0.1$	$1 \times 0.05 \times 0.05$	$0.1 \times 0.01 \times 0.01$
Both Ends Clamped or Free	*120 KHz* [77] (42)	*12 MHz* [7.7] (4.2)	*590 MHz* [380] (205)	*12 GHz* [7.7] (4.2)
Both Ends Pinned	*53 KHz* [34] (18)	*5.3 MHz* [3.4] (1.8)	*260 MHz* [170] (92)	*5.3 GHz* [3.4] (1.8)
Cantilever	*19 KHz* [12] (6.5)	*1.9 MHz* [1.2] (0.65)	*93 MHz* [60] (32)	*1.9 GHz* [1.2] (0.65)

"Nanoelectromechanical Systems", M. L. Roukes, Technical Digest of the 2000 Solid-State Sensor and Actuator Workshop, Hilton Head Isl., SC, 6/4-8/2000.

But: Mechanical energy vs. Thermal energy scaling …
Is there another way to get high frequencies?

Bulk vs. flexural modes

- We can also excite non-flexural modes: bulk modes!



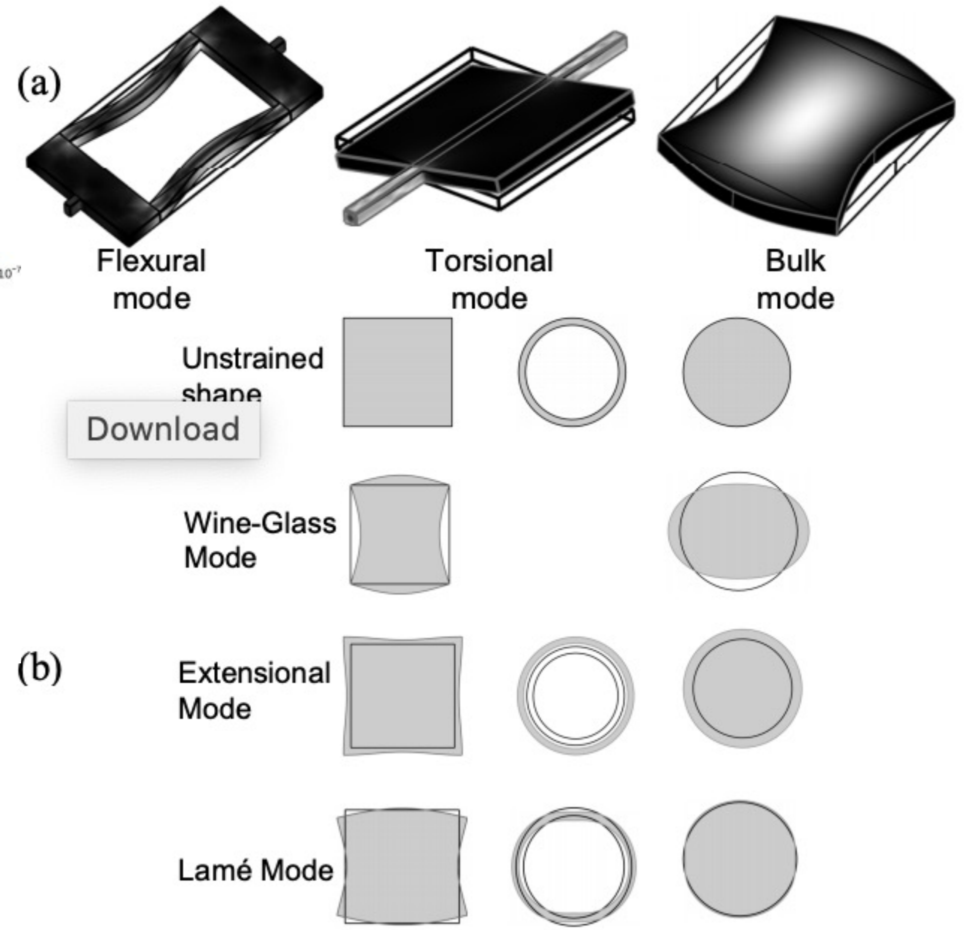


Figure 1: (a)Three basic types of resonators: Flexural, Torsional and Bulk mode structures (b) Commonly used Bulk mode resonator designs and various mode shapes

- Stored energy depends on moving mass: for bulk mode can make a thick and wide beams

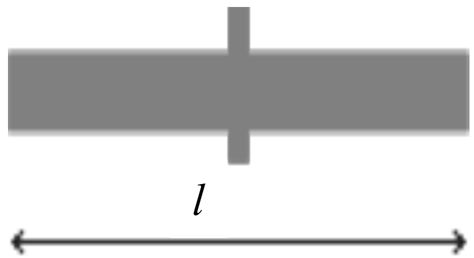
- Bulk mode = Very stiff: get high frequency

S. A. Chandorkar, M. Agarwal, R. Melamud, R. N. Candler, K. E. Goodson, and T. W. Kenny, "Limits of quality factor in bulk-mode micromechanical resonators," in *2008 IEEE 21st International Conference on Micro Electro Mechanical Systems*, doi: 10.1109/MEMSYS.2008.4443596.

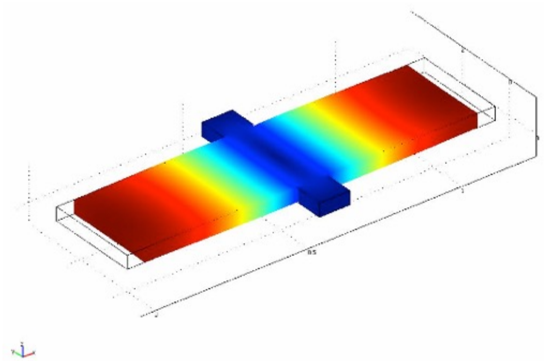
With

Bulk modes: example of suspended bar

- Suspended bar, exciting longitudinal (not bending) vibrations, central suspension



- Fundamental mode: $f_0 = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$ $\propto L^{-1}$
  - Independent of thickness!
  - Independent of width!



- Example, Al beam with l= 60 µm, E=85 GPa, ρ=4500 kg/m^3

  f_1= 36 MHz f_3=108 MHz

- Equivalent model

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k^*}{m^*}} \quad \propto L^{-1}$$

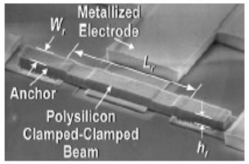
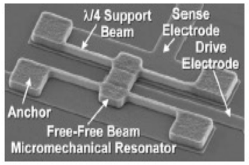
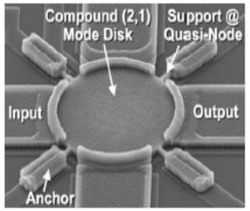
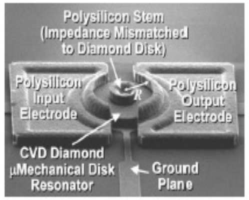
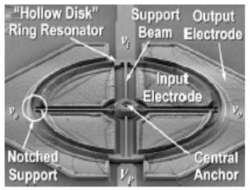
$$m^* = \rho \cdot w \cdot h \cdot \frac{l}{2} = m/2 \quad \propto L^3$$

$$k^* = \frac{\pi^2 Ewh}{2l} \quad \propto L^1$$

Evolution of MEMS resonators to bulk modes

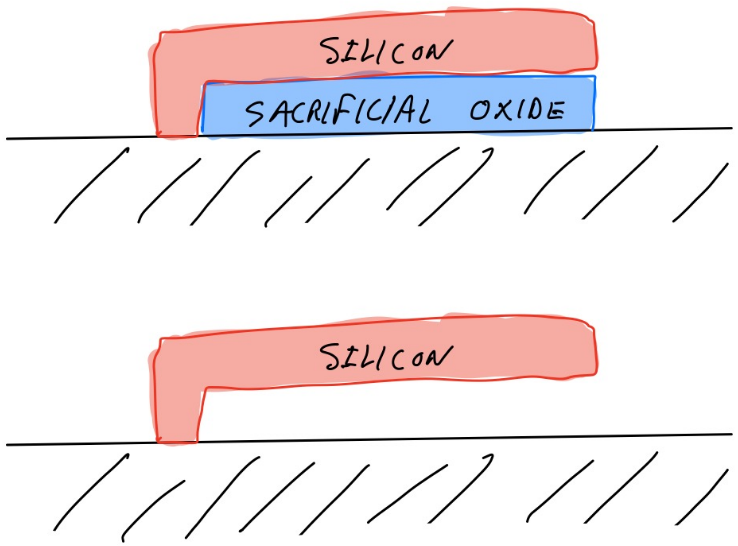
- Clark T.-C. Nguyen, “MEMS Technology for Timing and Frequency Control”, IEEE transactions on ultrasonics, ferroelectrics, and frequency control, Feb. 2007, p. 251

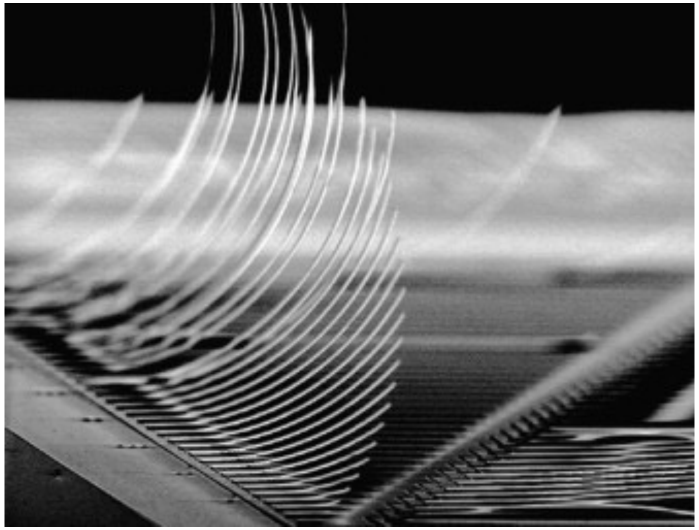
High Frequency Q Product Vibrating RF MEMS Devices.

Row	Resonator Type and Description	Photo	Performance
1	Clamped-Clamped Beam [19]: Flexural-mode beam fixed to the substrate at both ends. Micron-scale (i.e., 2-μm-thick) version is simple and works well below 30 MHz. Anchor losses reduce Q as frequency increases beyond 30 MHz. At right: 40-μm-long 7.8-MHz beam.		Demo'ed: $Q \sim 8{,}000$ @ 10 MHz (vac) $Q \sim 50$ @ 10 MHz (air) $Q \sim 300$ @ 70 MHz (anchor diss.) Q drop w/freq. limits freq. range Series Resistance, $R_x \sim$ 5–5,000 Ω*
2	Free-Free Beam [20]: Beam supported at flexural-mode nodal locations by quarter-wavelength torsional supports that “virtually levitate” the device, suppressing losses to anchors. Q remains high in vacuum as frequencies increase past 100 MHz. At right: 14.3-μm-long 82-MHz beam.		Demo: $Q \sim 28{,}000$ @ 10–200 MHz (vac) $Q \sim 2{,}000$ @ 90 MHz (air) No drop in Q with freq. Freq. Range: >1 GHz; unlimited w/scaling and use of higher modes Series Resistance, $R_x \sim$ 5–5,000 Ω*
3	Wine-Glass Disk [21]: Disk vibrating in the compound (2,1) mode. Can use either a center stem or perimeter supports. With quarter-wavelength perimeter supports located at radial nodal locations, achieves the highest Q's of any VHF on-chip resonator. At right: 26.5-μm-radius 73-MHz disk with perimeter supports.		Demo'ed: $Q \sim 161{,}000$ @ 62 MHz (vac) $Q \sim 8{,}000$ @ 98 MHz (air) Perimeter support design nulls anchor loss to allow extremely high Q Freq. Range: >1 GHz w/scaling Series Resistance, $R_x \sim$ 5–5,000 Ω*
4	Contour-Mode Disk [6], [22]: Disk vibrating in the radial-contour mode supported by a stem located at its center nodal point. Use of a material-mismatched stem maximizes the Q, allowing this design to set the record in frequency-Q product for any on-chip UHF resonator at room temperature. At right: 10-μm-diameter 1.5-GHz (in 2nd mode vibration) CVD diamond disk.		Demo'ed: $Q \sim 11{,}555$ @ 1.5 GHz (vac) $Q \sim 10{,}100$ @ 1.5 GHz (air) Balanced design and material mismatching anchor-disk design nulls anchor loss Freq. Range: >1 GHz; unlimited w/scaling and use of higher modes Series Resistance, $R_x \sim$ 50–50,000 Ω*
5	Spoke-Supported Ring [7]: Ring supported by spokes emanating from a stem anchor at the device center. Quarter-wavelength dimensioning of spokes nulls losses to the stem anchor, allowing this design to achieve the highest Q's past 1 GHz of any on-chip resonator. At right: 51.3-μm-inner and 60.9-μm-outer radii ring that attains 433 MHz in its 2nd contour mode.		Demo'ed: $Q \sim 15{,}248$ @ 1.46 GHz (vac) $Q \sim 10{,}165$ @ 1.464 GHz (air) $\lambda/4$ notched support nulls anchor loss Freq. Range: >1 GHz; unlimited w/scaling and use of higher modes Series Resistance, $R_x \sim$ 50–5,000 Ω*

Effect of Stress and Stress Gradients in cantilevers

- Due to fabrication processes, nearly all micromachined cantilevers have both **stress** and **stress gradients**





Witvrouw, A., Tilmans, H. A. C. & De Wolf, I. Materials issues in the processing, the operation and the reliability of MEMS. *Microelectronic Engineering* **76,** 245–257 (2004).

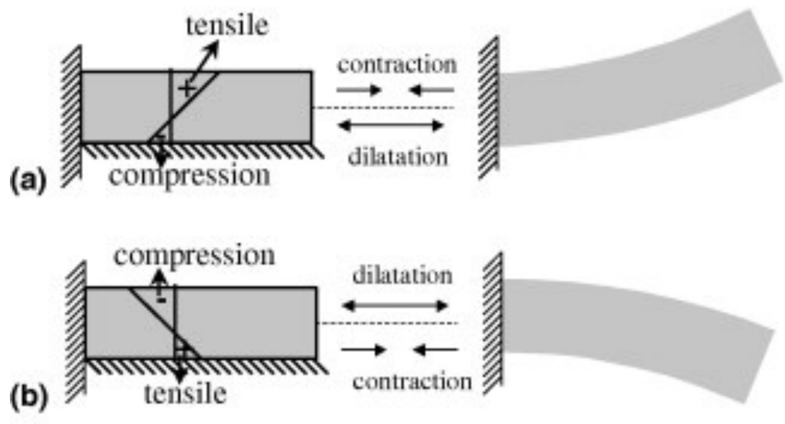
Stress and stress gradient in singly-clamped cantilever

- Effect of Stress gradients depends strongly on cantilever clamping conditions

- Simple cantilever (single suspension)

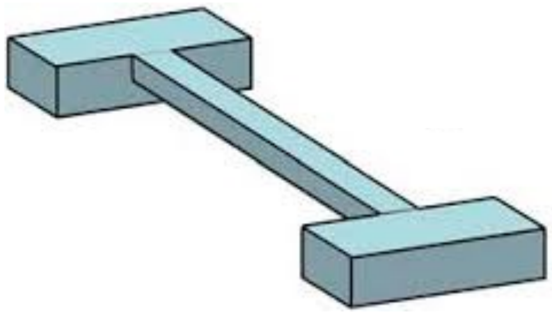
  - Stress: only expansion or contraction (like Temperature change)

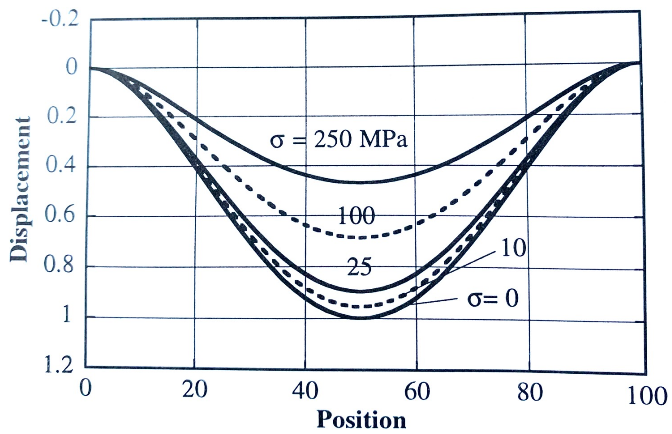
  - Stress gradient: large effect! Beam bending



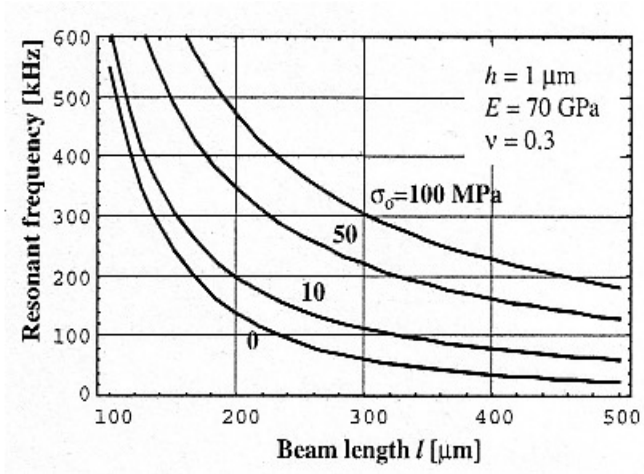
Stress and stress gradients: doubly-clamped cantilever

- Stress gradient
  - small effect
- Tensile stress
  - increase spring constant,
  - increase resonant frequency
- Compressive stress
  - buckling,
  - non-linear effective spring const.





Bending of a clamped silicon beam 100x2x2 μm³ under uniform load for different values of tensile stress.
S. Senturia, "Microsystem Design", p.232



Effect of tensile stress on the resonance frequency of doubly clamped cantilever

Standard test structures to measure stress in MEMS

- Guckel ring to measure tensile stress

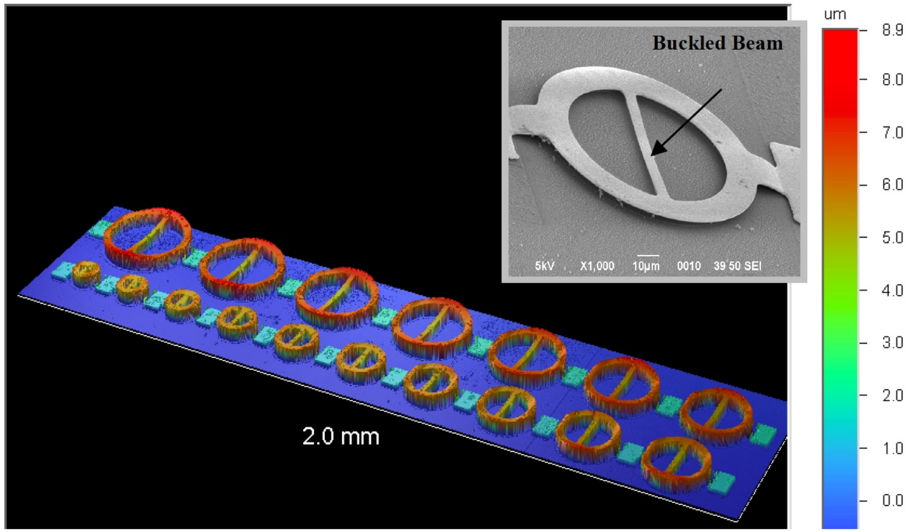
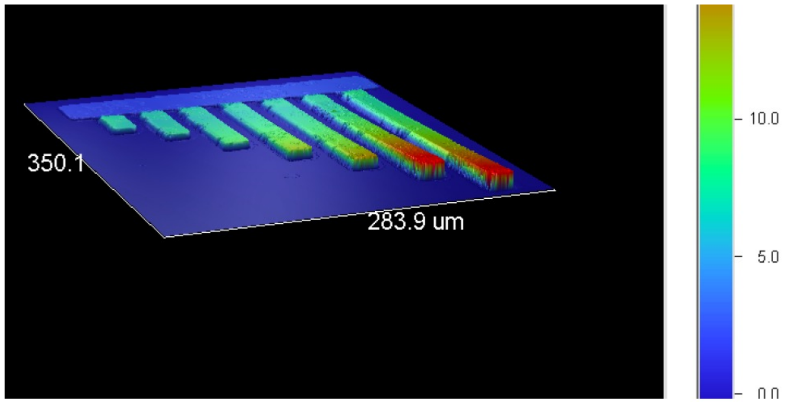
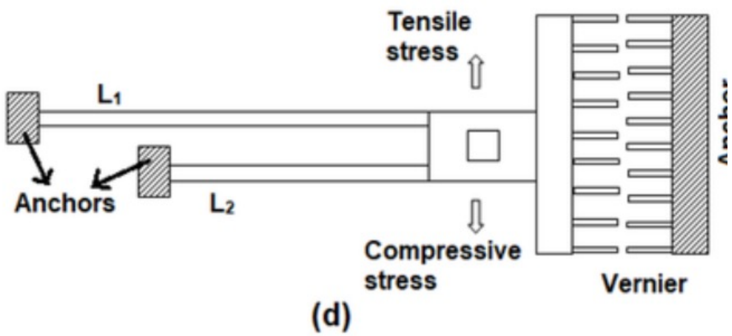


Fig. 7. 3D Optical profiler image of 2µm thick plated Au Guckel rings array with SEM micrograph of critical buckled central beam of R_c 60µm.

Sharma et al, Sensors & Transducers Journal, Vol. 13, 2011, pp. 21-30

- Cantilevers



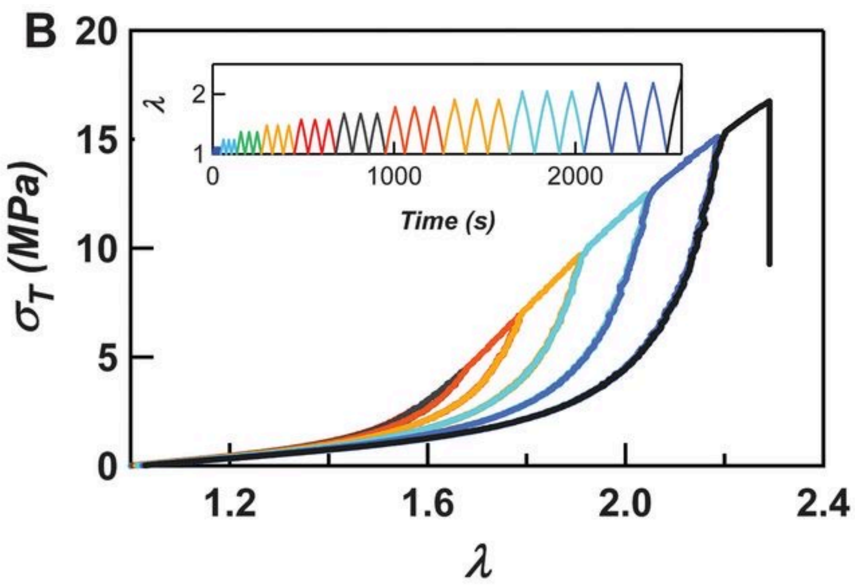


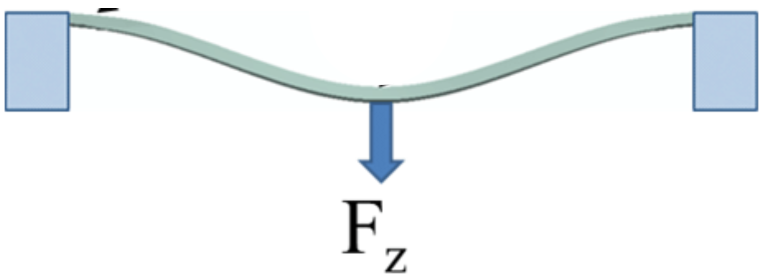
2021 paper: https://link.springer.com/article/10.1007/s10854-021-05405-8

3. Non-linear effects in MEMS cantilevers

Non-linearities:

1. **Material** and 2. **Geometrical**

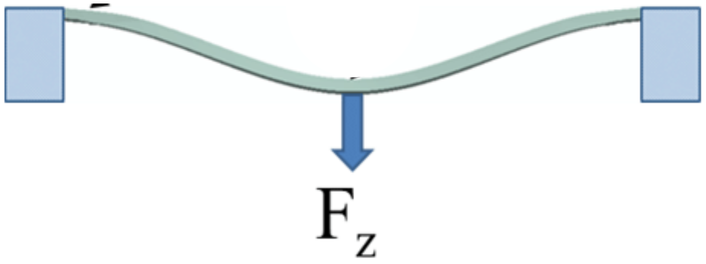




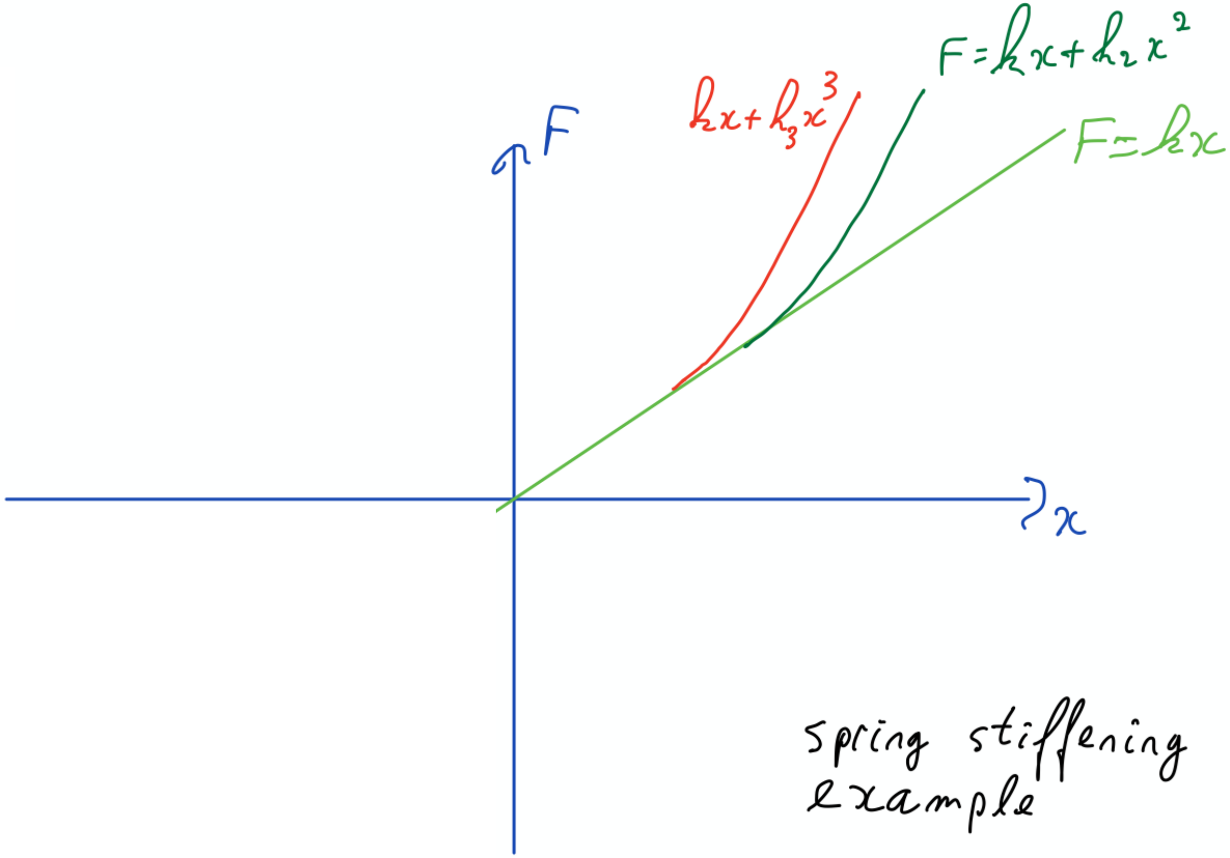
E Ducrot *et al*, Science 2014. DOI: 10.1126/science.1248494

Non-linear stress-strain
Hysteresis
Mullins effect…

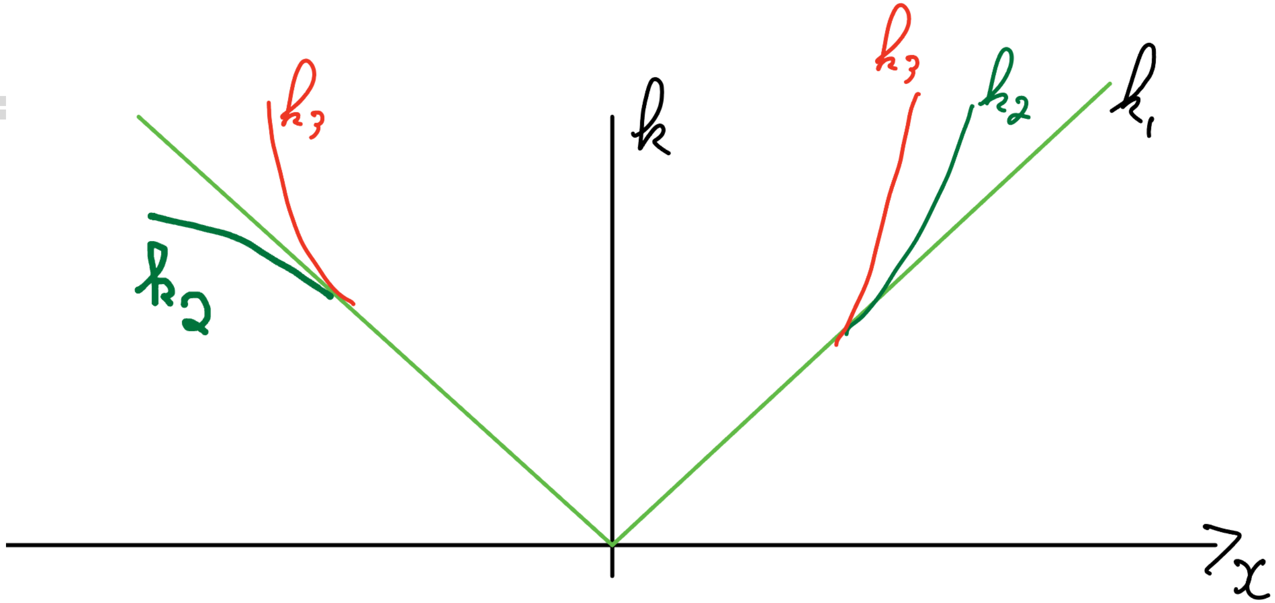
Spring constant for Geometrical non-linearities

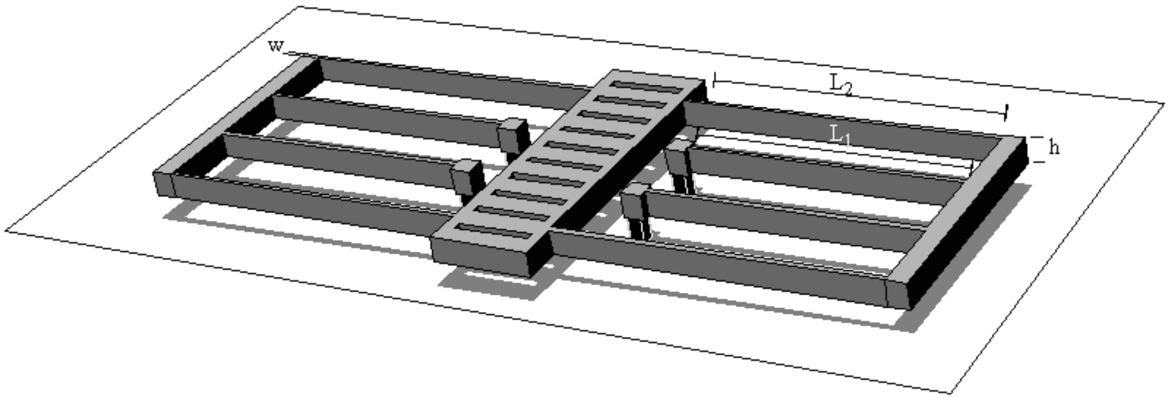


Bending → Bending + Stretching

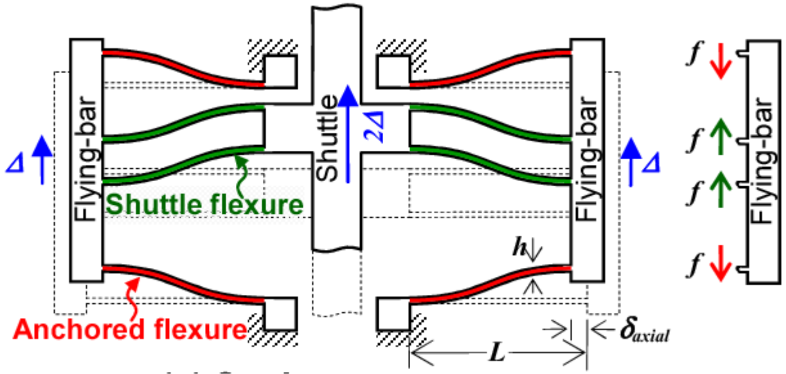


How to express the non-linearity? Tailor expansion?





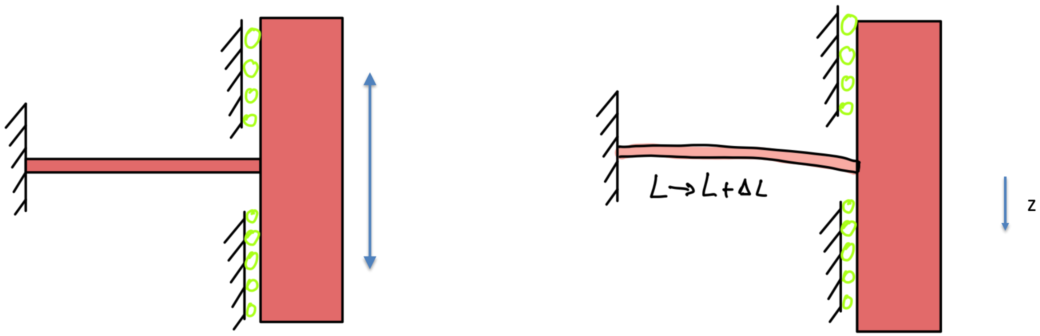
Standard Folded-Beam Suspension



http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7050925

Non-linear effects in fixed guided beam: Very Important effect in MEMS!

$$F_z = k \cdot z + k_3 \cdot z^3$$



$$k = \frac{Ew \cdot h^3}{l^3}$$

As mass is moved up or down, the spring bends… but it also stretches. So stiffness increases

$$k_3 \cong \frac{252 EA}{175 l^3} = 1.4 \frac{Ew \cdot h}{l^3} \propto \frac{1}{L}$$

Beam length L, thickness h, witdh w

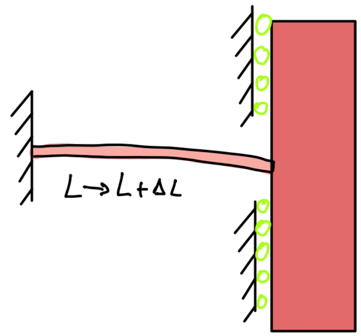
$$\frac{k_3}{k} = 1.4 \frac{1}{h^2} \propto L^{-2}$$

$$F_z = k \cdot z \left(1 + \frac{k_3}{k} z^2\right) = k \cdot z \left(1 + 1.4 \cdot \left(\frac{z}{h}\right)^2\right)$$

Non-linearity becomes more important as we scale down!

Non-linear effects in fixed guided beam

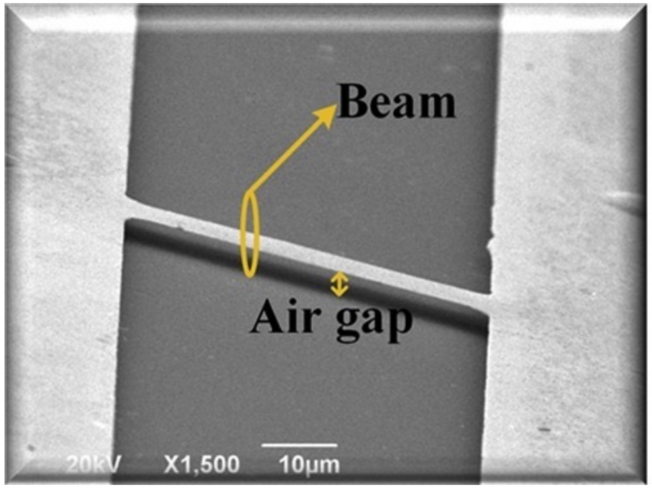
- important non-linearity when displacement *z* is a sizable fraction of thickness *h*
- a simple criterium: non-linear (10% effect) behavior when $z > 0.4h$
- Important consequences for MEMS resonators:
  - frequency stability: f_{res} depends on amplitude!
  - Limits power handling



$$k_3 \cdot z^3 > 0.1 \cdot k \cdot z$$

10%

When $z > 0.4h$



20 cm thick

40% motion = 8 cm

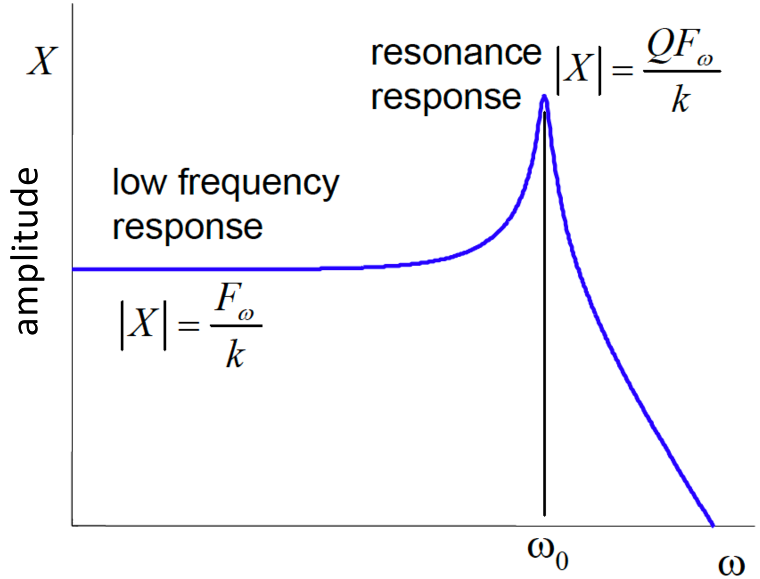
1 μm thick

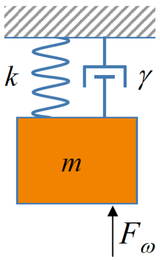
40% motion = 400 nm

Oscillator with non-linear spring: Duffing equation

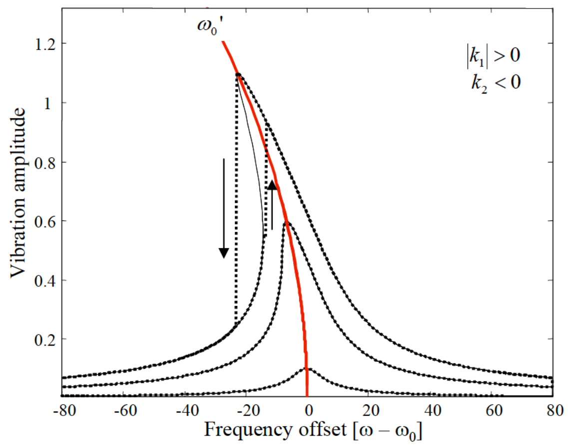
Good overview of Duffing eq. for MEMS:
http://www.kaajakari.net/~ville/research/tutorials/nonlinear_resonators_tutorial.pdf

$$\ddot{x} + 2\lambda\dot{x} + \omega_0{}^2 x + k_3 x^3 = F(t)$$

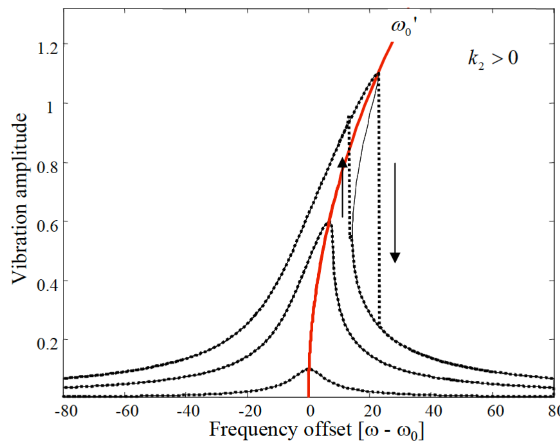




Linear case $k_3=0$



Spring softening (will return to this in electrostatic chapter)



Spring stiffening (mechanical)

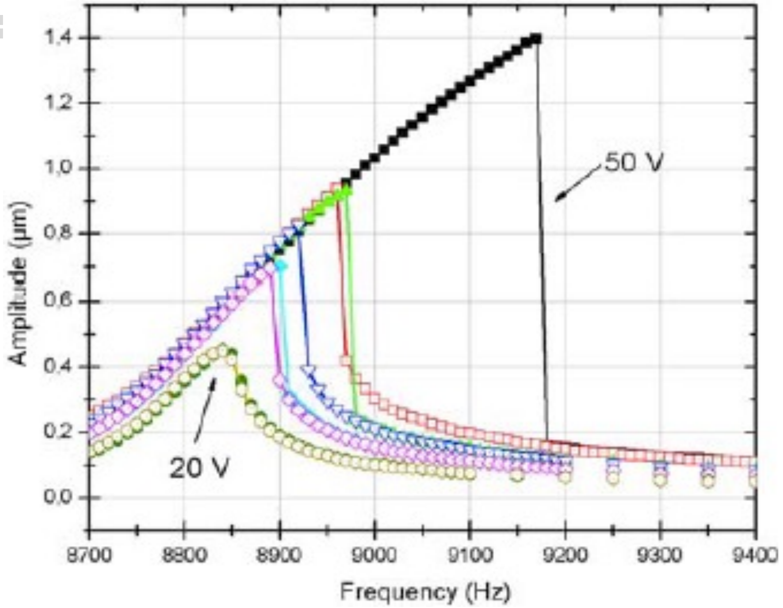
Duffing equation

- Duffing equation it has two solutions: One when sweeping frequency down and one when sweeping up !

- approximatefrequency shift for small amplitudes (z: displacement, h: thickness):

$$\frac{\omega^*}{\omega_0} = \sqrt{\frac{k^*}{k}} \approx \sqrt{1+1.4\cdot\left(z/h\right)^2} \qquad \frac{\Delta\omega}{\omega_0} \approx 0.7\cdot\left(z/h\right)^2$$

for 1% *h* displacement, 0.007 % relative frequency shift
for 10% *h* displacement (0.5 µm), 0.7 % relative frequency shift (do à do-dièse)



Test device: h=5µm, l=500µm, f_0= 8800
R. Guerre, EPFL-LMIS4

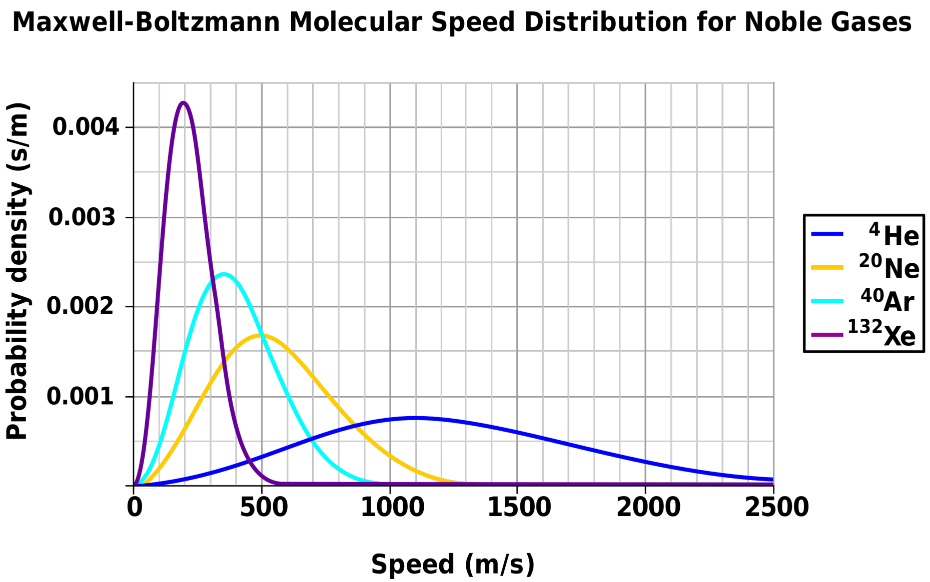
- Hysteresis rule: no hysteresis as long as:

$$|x| < \sqrt{\frac{8k}{3k_3 Q}} \approx 0.7\cdot h\sqrt{\frac{1}{Q}}$$

for Q=10 x_{max}=0.22
only small displacements allowed for high Q devices

4. Thermo-Mechanical noise

Equipartition theorem: ½ k_BT for each DoF.



https://en.wikipedia.org/wiki/Equipartition_theorem

Probability density functions of the molecular speed for gases at 25° C

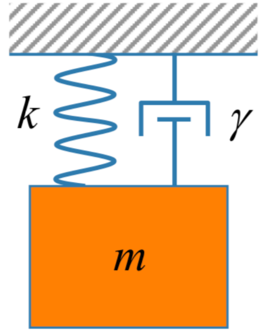
Ideal gas

$$v_{\rm rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_BT}{m}}$$

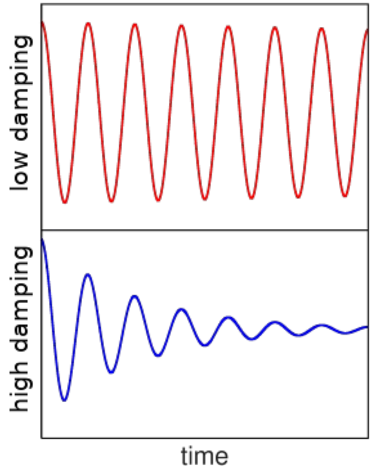
k_BT = noise power density in (W/Hz)

- How much does a MEMS cantilever weigh compared to a gas molecule?
- Is that a fair comparison?

Thermo-Mechanical noise



- Simple System = mass+ spring+ damper
- How is the mass thermally coupled to the world? Via damping element
- Damping = coupling
- No damping = mass decoupled from environment (no thermal exchange, no thermal noise, infinite Q)
- High damping = mass strongly coupled to the environment (lots of thermal exchange, high thermal noise, low Q)



- May seem a bit counter-intuitive: more damping, more thermal noise! $Q \sim L$
  - the issue is what frequency we consider: at resonance, or below resonance ???

Thermo-Mechanical noise

- Simple Harmonic oscillator exposed to a random fluctuating force F_n:

$$F_n(t,\lambda) = m\ddot{x} + \lambda\dot{x} + kx$$

λ=damping coefficient $\lambda = \frac{\omega_0 m}{Q} = \frac{\sqrt{k.m}}{Q} \sim L$

- Spectral density of fluctuating force F_n: (force-voltage analogy) $F_{n,rms}(\omega) = \sqrt{4k_bT\lambda}$ $\sim L^{0.5}$

  $[N/\sqrt{Hz}]$

  - **The damping coefficient is crucial for allowing energy exchange between the oscillator and the medium** (both for coupling energy in and out of the oscillator)

  - Any mechanical system can be analyzed for mechanical-thermal noise by adding a force generator at each damping element.

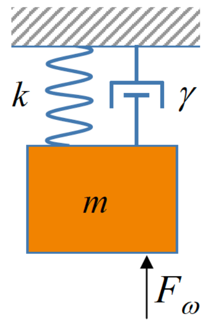
Good derivation in : http://www.kaajakari.net/~ville/research/tutorials/mech_noise_tutorial.pdf

Thermo-Mechanical noise, below the resonance frequency

- Position fluctuations (at frequencies well **below the resonance frequency**):

$$F = k \cdot x \longrightarrow x_{n,rms}(\omega) = \frac{F_{n,rms}(\omega)}{k} \qquad \sim L^{-0.5}$$

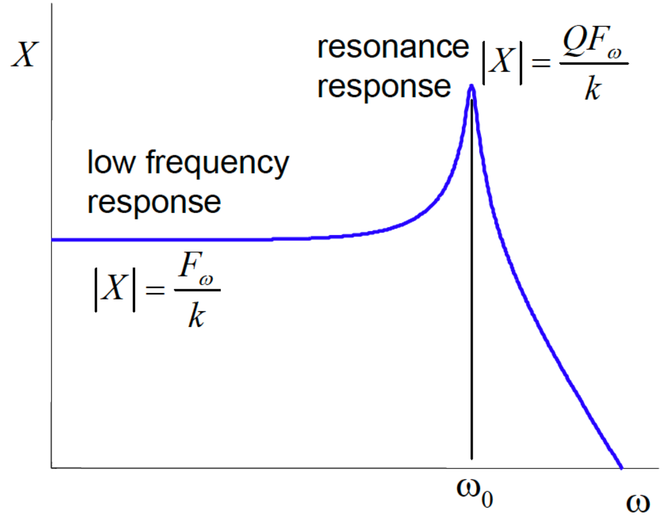
$$x_{n,rms}(\omega) = \frac{\sqrt{4k_bT\lambda}}{k} \quad \left[m/\sqrt{Hz}\right]$$ Spectral density



- **Mean** thermal displacement **for ω<ω$_0$** is:

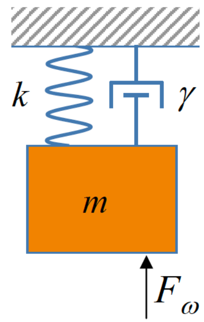
$$x_{n,rms}\Big|_{f_1}^{f_2} = \frac{\sqrt{4k_bT\lambda}}{k} \cdot \underbrace{\sqrt{\Delta f}}_{\text{bandwidth}} \qquad [m]$$

$$\sim L^{-1.5}$$



Thermo-Mechanical noise in accelerometer

$$S_x = \frac{1}{\omega_0^2}$$



- Displacement due to external acceleration $x_a = \frac{ma}{k}$
- Minimal detectable acceleration a_{min} due to thermal noise: $a_{min} = \sqrt{\frac{4k_b T\lambda}{m^2}}\sqrt{\Delta f}$
  ie when $x_a = x_{noise,\,rms}$
- Scaling (assuming $\lambda \propto L$)

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \propto L^{-1} \qquad Q = \frac{\omega_0 m}{\lambda} \quad \propto L \qquad a_{min} = \sqrt{\frac{4k_b T\omega_0}{mQ}}\sqrt{\Delta f} \qquad a_{min} \propto L^{-5/2}$$

Typically a few tens of µg/sqrt (Hz) to mg/sqrt (Hz)

- The miniaturization of accelerometers dramatically reduces sensitivity (due to thermal noise)!
- The resolution can be improved by increasing Q, lowering ω_0 or increasing mass
- High Q element is however not desirable because it induce long oscillatory tail in signals …
- Same reasoning for pressure sensors, microphones, etc.

Key paper: T. Gabrielson. "Mechanical-thermal noise in micromachined acoustic and vibration sensors". IEEE Trans. on elec. Dev. Vol. 40. 5 (1993)

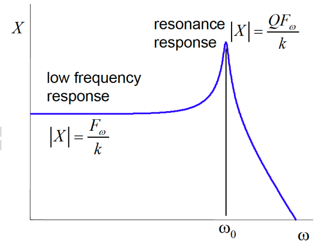
- Example (below resonance)

  - Si MEMS inertial mass 100 µm thick, 200 µm x 100 µm area
  - ω_0=20 kHz
  - Q =1
  - 1000 Hz bandwidth

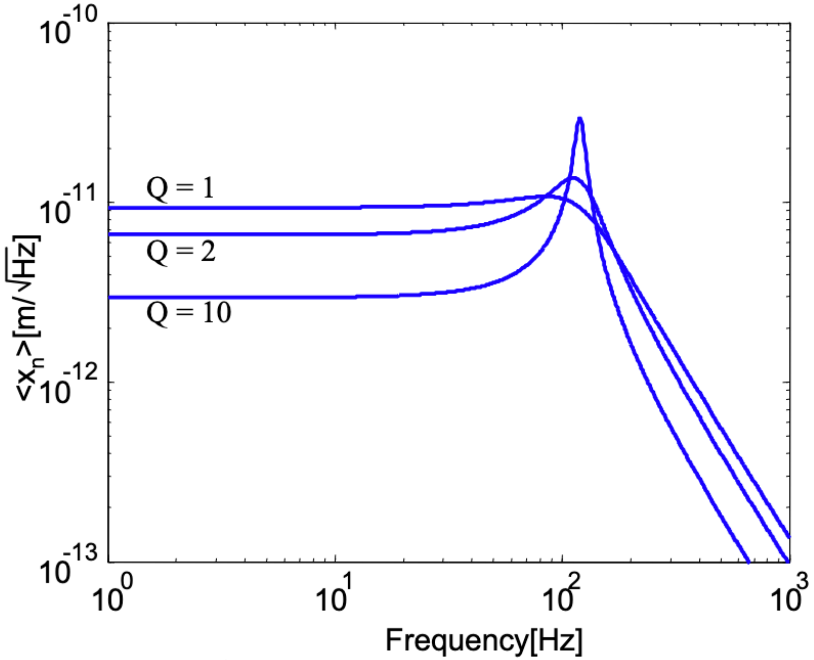
$$a_{\min} = \sqrt{\frac{4k_b T\omega_0}{mQ}}\sqrt{\Delta f}$$

- a_{min} = 10^{-2} m.s^{-2}
  = 1 milli-"g"

Thermo-Mechanical noise vs Q



https://www.kaajakari.net/~ville/research/tutorials/mech_noise_tutorial.pdf



$$< x_n > = \sqrt{\overline{x_n^2}}$$

$$x_{n,RMS}\Big|_{f_1}^{f_2} = \frac{\sqrt{4k_BT}}{k}\,\frac{\sqrt{\omega_0 m}}{\sqrt{Q}}\cdot\sqrt{\Delta f}$$

Below Resonance

Device dimensions: 1 mm x 1 mm x 0.2 mm
mass: $4.4 \cdot 10^{-7}$ kg
and spring constant: 0.25 N/m.

Quality Factor scaling

The quality factor is defined as:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

Q for many loss factors scales as L

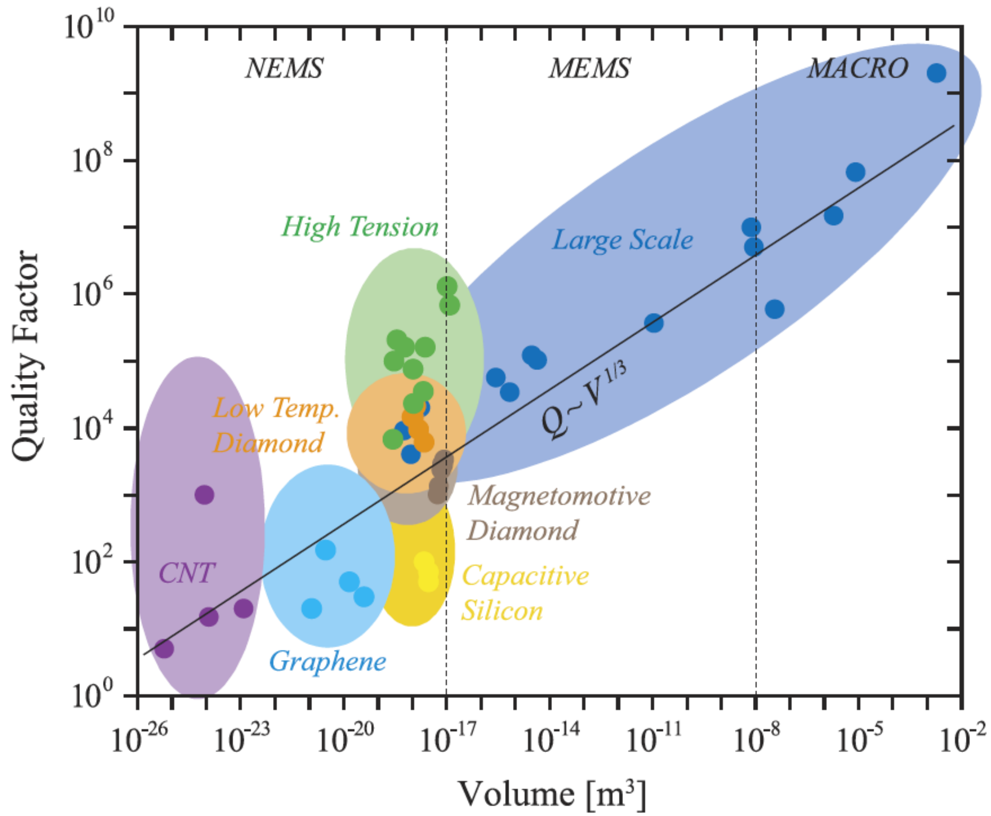
Q factor depends on:

$$\frac{1}{Q} = \frac{1}{Q_1} + \frac{1}{Q_2} + \cdots = \sum_j \frac{1}{Q_j}.$$

- Externally
  - Air damping
  - Clamping losses at supports
  - Coupling losses from transducers

- Internally
  - Thermo-elastic effects from defects in bulk, interfaces, fab-related damage, adsorbates on surface, …

○ Q often depends on *surface to volume ratio* $\propto L$



Imboden, M. & Mohanty, P.
Dissipation in nanoelectromechanical systems
Physics Reports **534,** 89–146 (2014).

5. Surface Forces in MEMS

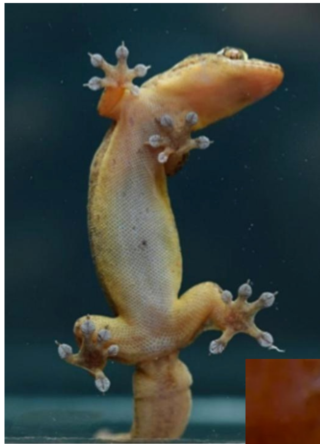
Surface Forces in MEMS

surface scales with L^2

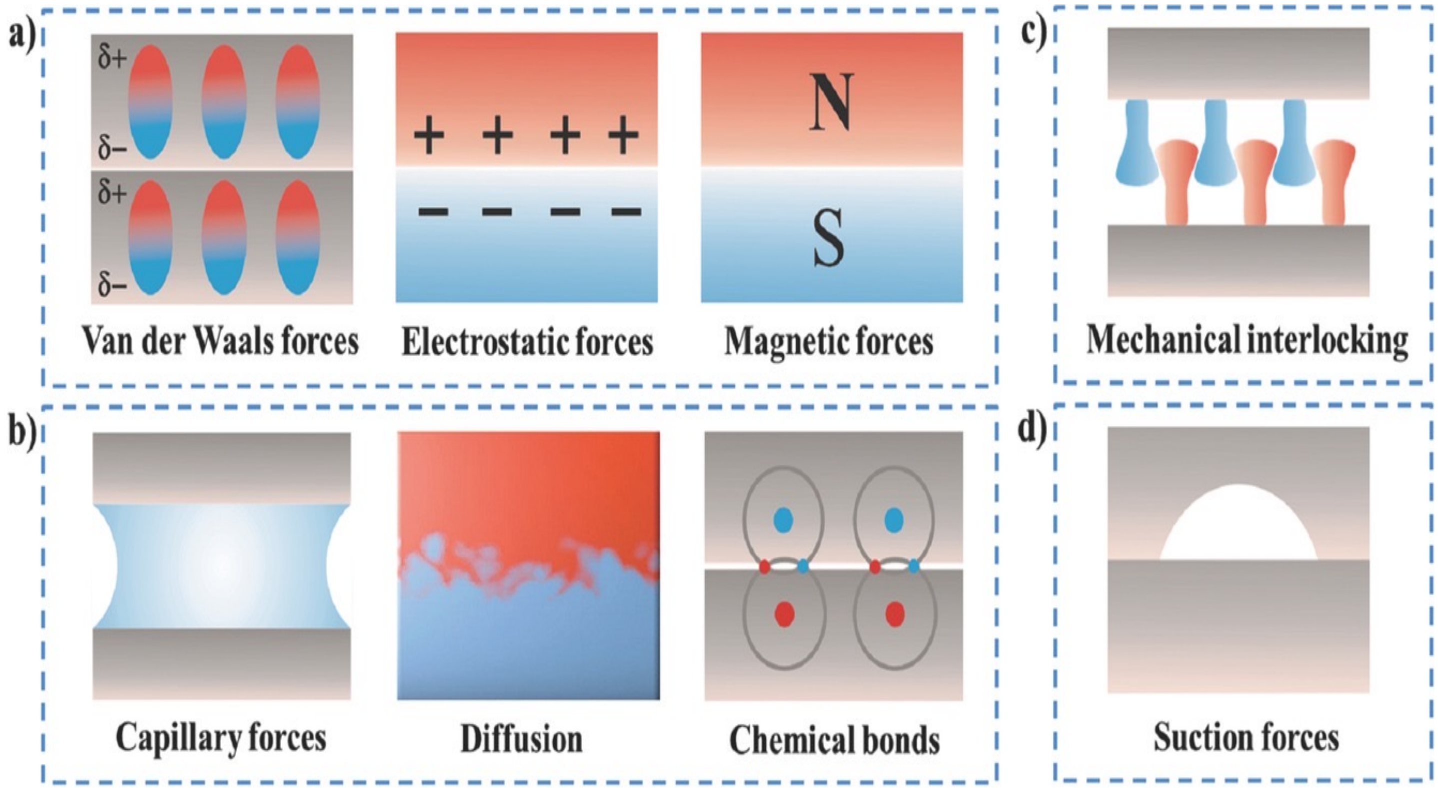
- **Non contact**
  - Surface charges
  - Van der Waals
  - Casimir

- **Contact**
  - Capillary
  - Adhesion

- **Effect of surface forces:**
  - Collapse
  - Sticking
  - Effective friction coefficient



Main surface interactions



Z. Gu et al, 2016. Adv. Science, vol 3, 1500327
https://onlinelibrary.wiley.com/doi/10.1002/advs.201500327

EPFL

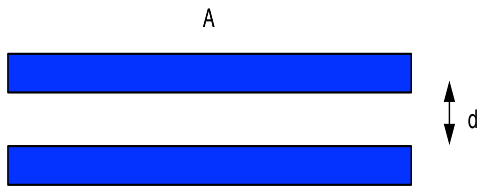
Electrostatic Adhesion Forces

Charges trapped on surfaces:

- Charges on a particle or surface produce a classical Coulomb attraction

$$F_{es} = -\frac{1}{2}\frac{Q^2}{C \cdot d} = -\frac{1}{2}\frac{\sigma_s{}^2 A^2}{C \cdot d} = -\frac{1}{2}\frac{\sigma_s{}^2 A}{\varepsilon_0}$$

Constant charge density σ_s



- **For a constant surface charge, the force is independent of the distance** !
  - Eg with σ_s ~ 10^{-5} C/m^2 $\quad p_{es} = \frac{F_{es}}{A} = \frac{\sigma_s{}^2}{2\varepsilon_0}$ (~0.00006 atm)

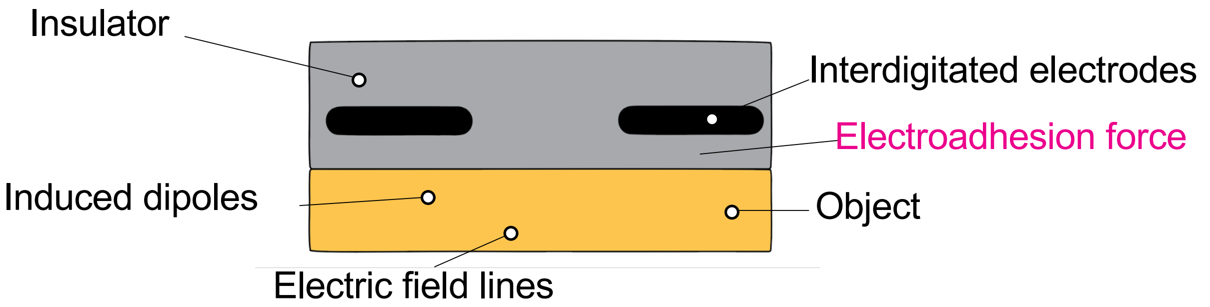
- Electrostatic image forces are important only for materials that can carry (trap) surface charge density, such as polymers with low conductivity or other good insulators.

Long-range

Electroadhesion: from discovery to applications

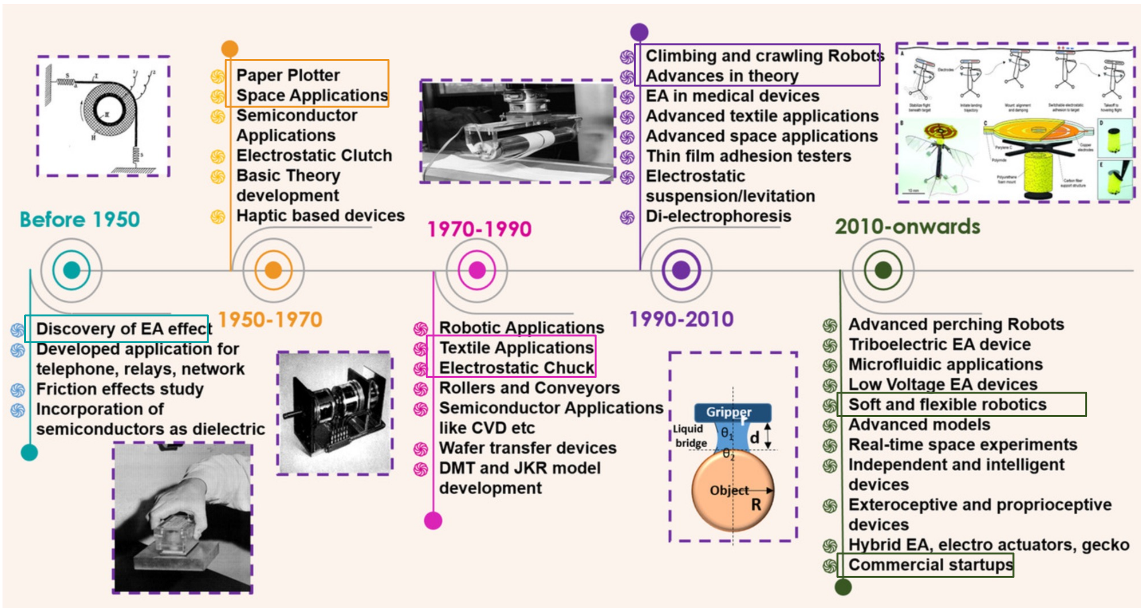
EPFL

Electroadhesion working principle



$$\sigma_{EA} \sim \frac{V^2}{t^2} \; \varepsilon_{BL}^2$$

Milestone achievements in the last 100 years [1]

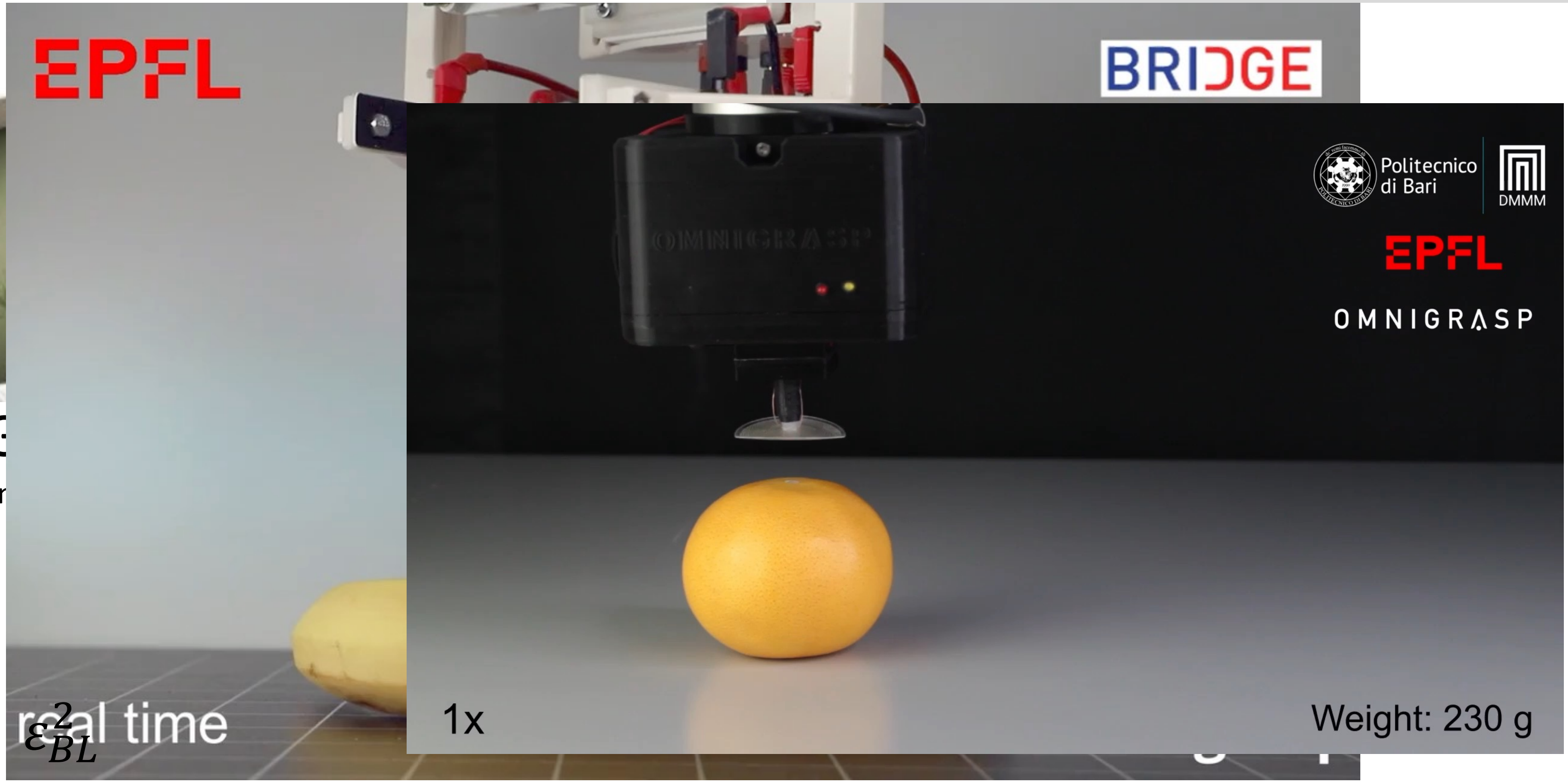


[1] R. Pandey, et al. *Advanced Intelligent Systems* 4.7 (2022)

Electroadhesion: high shear force due to high normal force

SRI, later G

https://grabitinc.cor



$$\sigma_{EA} \sim \frac{V^2}{t^2} \ \varepsilon_{BL}^2$$

V. Cacucciolo, H. Shea, G. Carbone, Peeling in Electroadhesion Soft Grippers, Extreme Mechanics Letters 2022

Range: 100s of µm

Long-range

Van der Waals forces

Van der Waals forces (or London dispersion forces) are due to induced dipole interactions from neighboring molecules. These interactions are described by the Lennard-Jones energy potential:

$$w_{LJ}(r) = -\frac{A}{x^6} + \frac{B}{x^{12}}$$

The force is the derivative of the potential with distance:

$$F_{vdw} \propto \frac{1}{x^7}$$

Hamacker calculation for the force between a plane and a sphere of radius r:

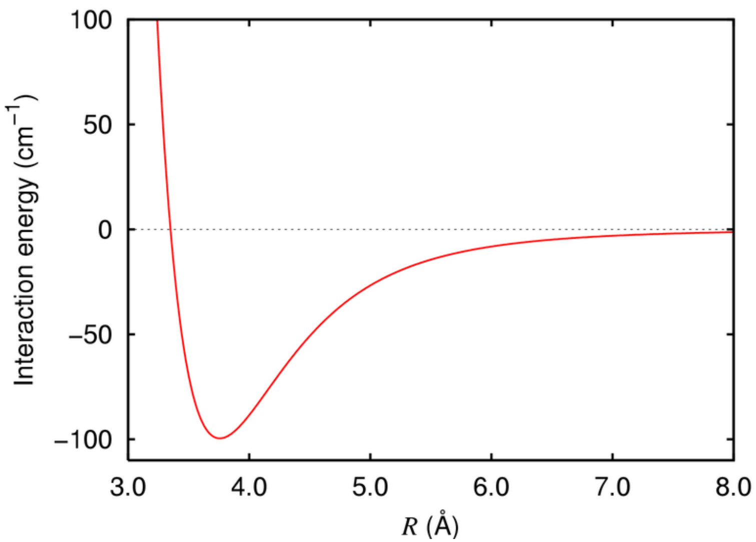
$$F_{vdw} = \frac{H \cdot r}{6 \cdot x_0^{\,2}}$$

Typical values H=10^{-19} J and x_0=0.4 nm

H is the Hamacker constant (unit: Joules), related to surface energy

r: radius> x_0,

x_0: equilibrium “contact” spacing (typically 0.3 nm to 0.5 nm)



Extremely Short-range

Van der Waals forces

The force required to overcome the van der Waals attraction of a perfectly rigid round r = 0.5 µm particle to a diamond plate in vacuum (with d_0 = 0.4 nm, H = 3.4 10^{-19}J) is:

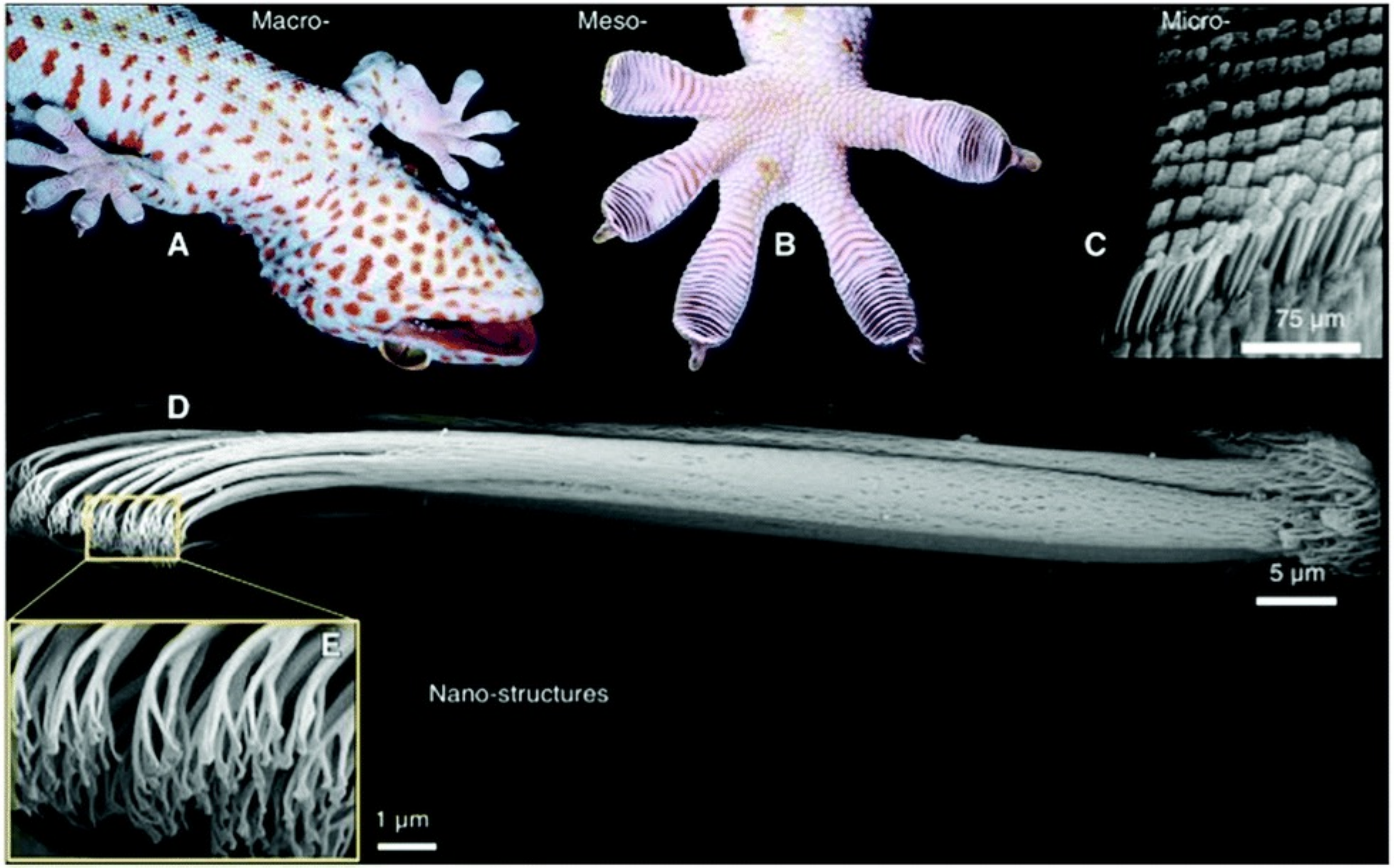
F_{vdW} ~ 180 nN. $F_{gravity}$ ~ 10^{-7} nN.

The Johnson–Kendall–Roberts theory adhesion force for a sphere-plane model also provides an estimate of van der Waals force:

$$F_{vdw} = \frac{3}{2} \cdot \pi \cdot r \cdot \gamma$$

typical surface energy for silica surfaces γ = 50 mJ/m^2

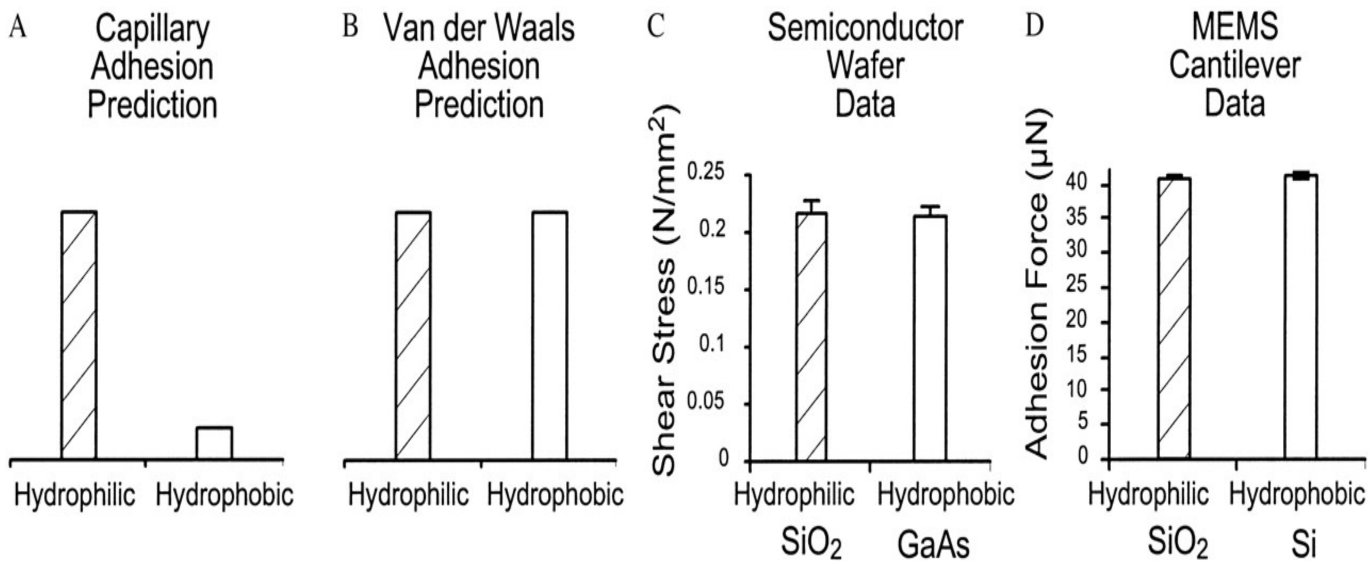
How Geckos walk on the ceiling



Adhesive function of the synthetic is similar to that of natural gecko setae, suggesting that specific surface chemistry is not required, and that an array of small, simple structures can be an effective adhesive.

K. Autumn et al., "Evidence for van der Waals adhesion in gecko setae", PNAS, 2002, vol. 99, no. 19, 12252-12256

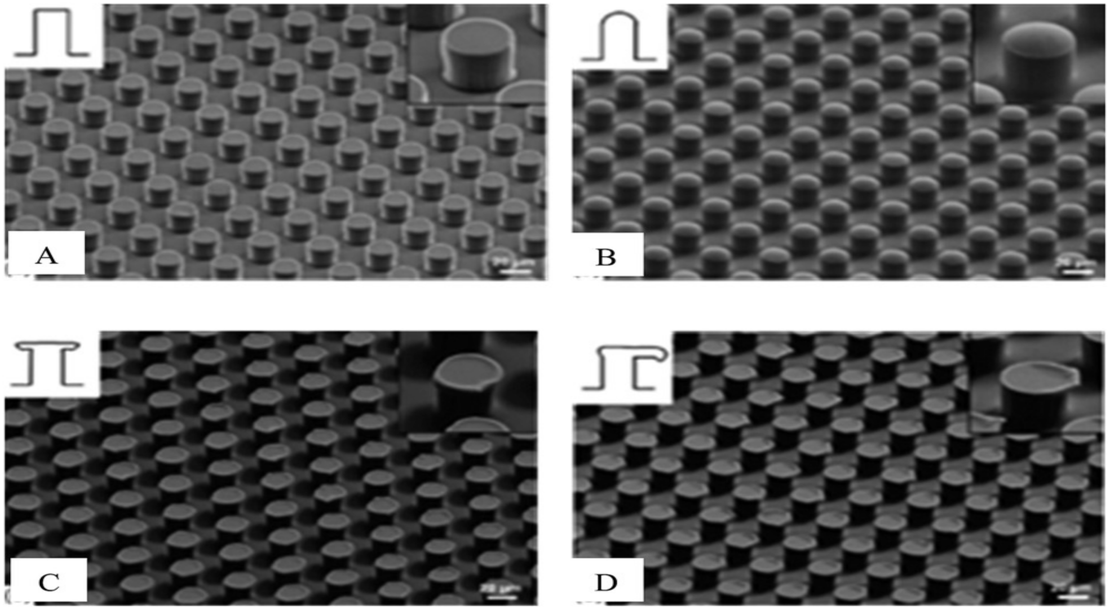
Van der walls or capillary forces?



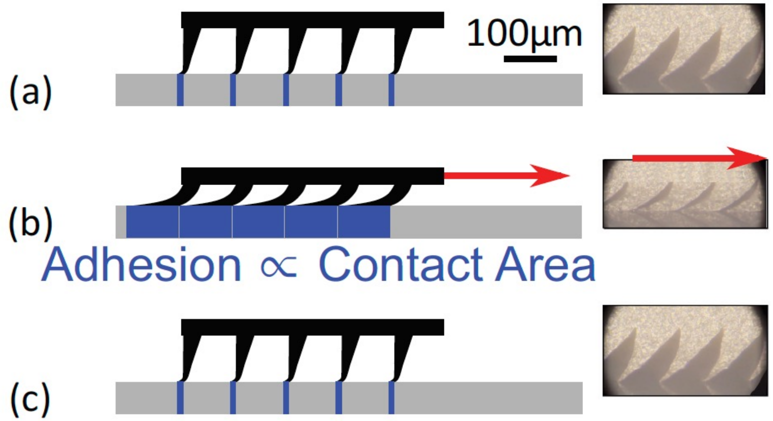
- Gecke foot Adhesion to hydrophilic and hydrophobic polarizable surfaces is similar.
- Therefore, one can reject the hypothesis that wet, capillary interactions are necessary for gecko adhesion in favor of the van der Waals hypothesis

K. Autumn et al., “Evidence for van der Waals adhesion in gecko setae”, PNAS, 2002, vol. 99, no. 19, 12252-12256

Dry adhesive: artificial Gecko features

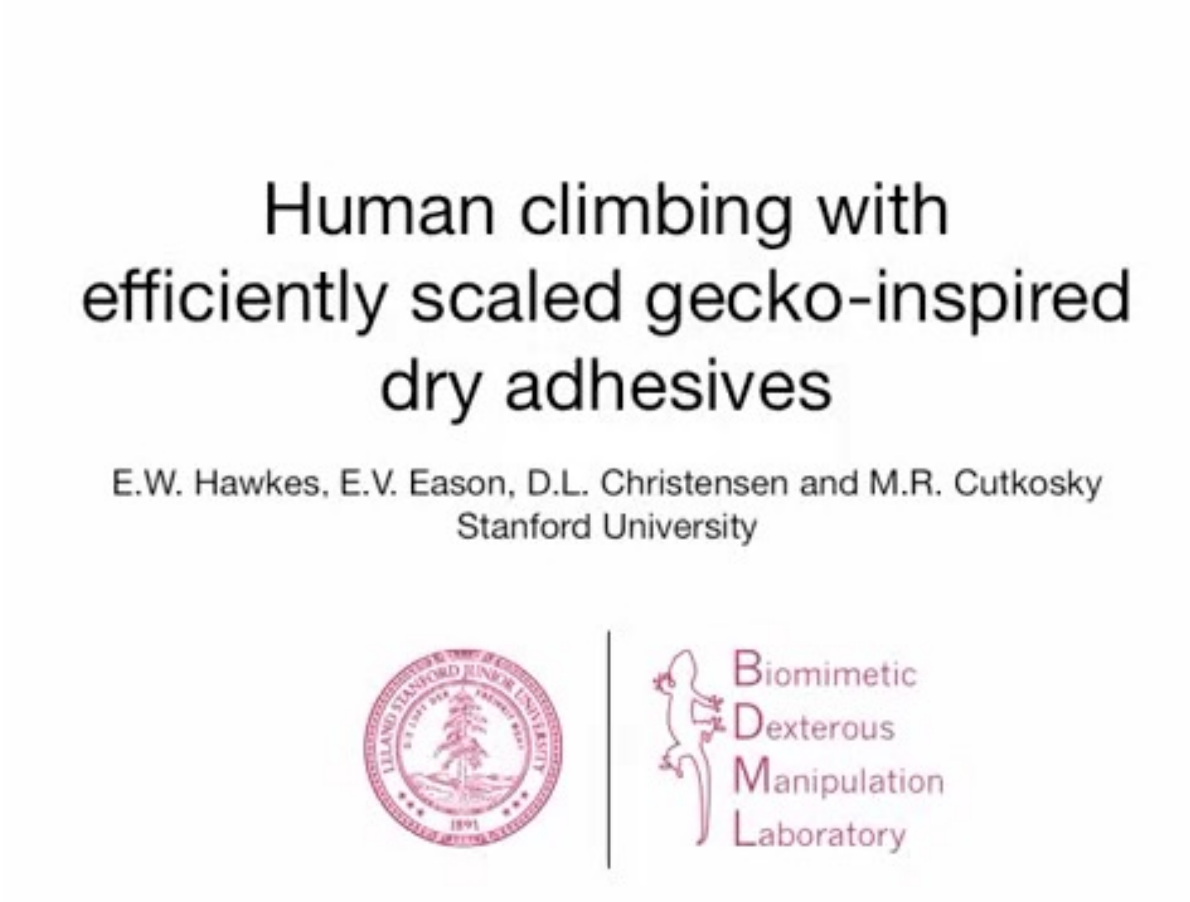


Sahay, R., et al. (2015). “A state-of-the-art review and analysis on the design of dry adhesion materials for applications such as climbing micro-robots“
RSC Adv., *5*(63), 50821–50832. doi:10.1039/C5RA06770G



Hawkes, E. W., Jiang, H. & Cutkosky, M. R. Three-dimensional dynamic surface grasping with dry adhesion. *The International Journal of Robotics Research* 16–16 (2015). doi:10.1177/0278364915584645

Human geckos



Hawkes et al, Human climbing with efficiently scaled gecko-inspired dry adhesives. *Journal of the Royal Society Interface* **12,** 20140675–20140675 (2015).

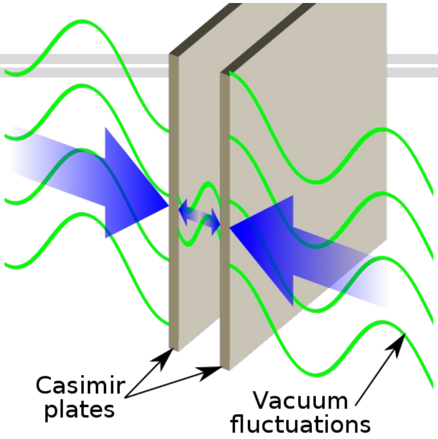
Casimir force

- Casimir forces are interactions between electrically neutral conductive materials.
- Striking manifestation of quantum fluctuations.
- The boundary conditions imposed on the electromagnetic fields lead to a spatial redistribution of the mode density with respect to free space, creating a spatial gradient of the zero-point energy density and hence a net force between the metals.
- Between two parallel metal plates, the Casimir force is attractive and is given by:

$$F_c = \frac{\hbar c \pi^2}{240} \cdot \frac{A}{d^4} = 1.3 \cdot 10^{-27} \cdot \frac{A}{d^4} \quad \propto d^{-4}$$

- *d*: distance between surfaces, ħ is the Plank constant, A: area of surfaces, c: speed of light
- *a* correction factor is necessary for non ideal conductors and surfaces

- Casimir forces are inherently mesoscopic in nature: only substantial values when the separation between the metallic surfaces is <100 nm (for 10 nm spacing: 1 bar pressure)

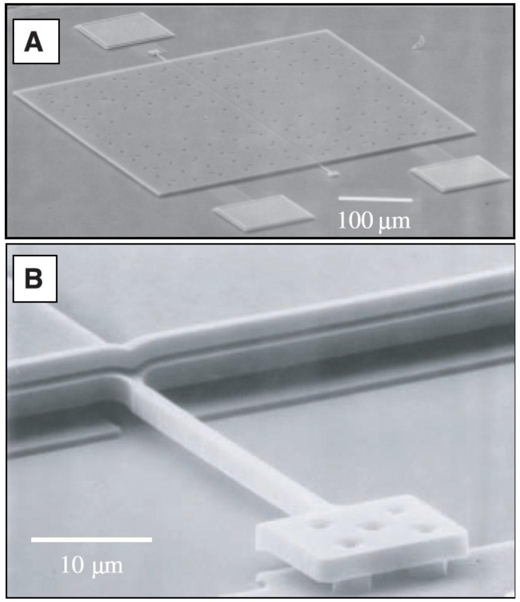


https://en.wikipedia.org/wiki/Casimir_effect

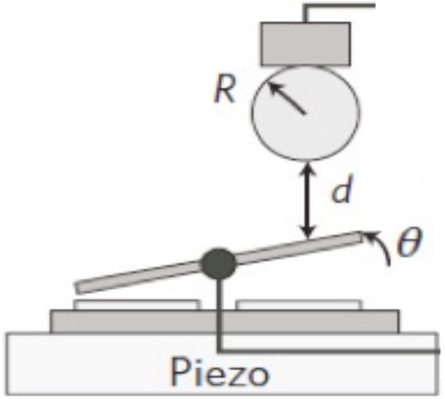
Interesting water wave analogy

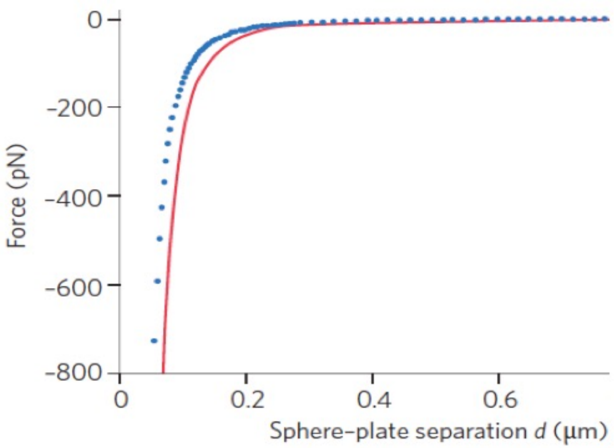
Short-range

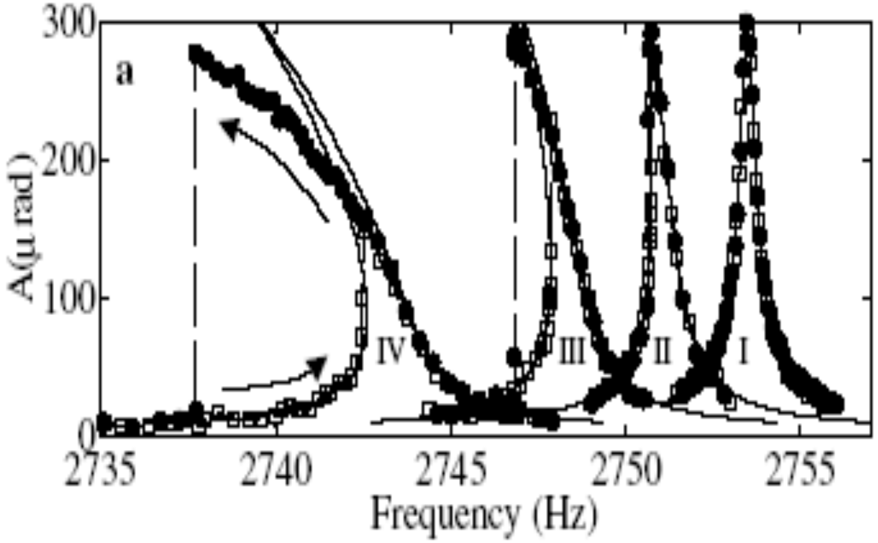
Casimir force & non-linear oscillators



Chan, H. B., et al, Quantum mechanical actuation of microelectromechanical systems by the Casimir force. *Science* **291,** 1941–4 (2001).





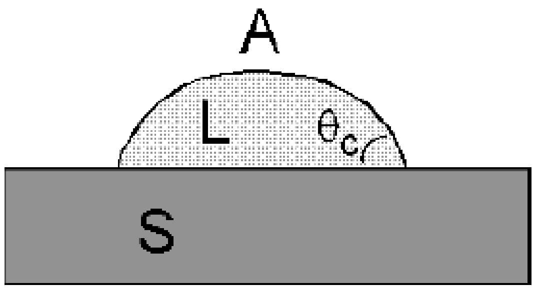


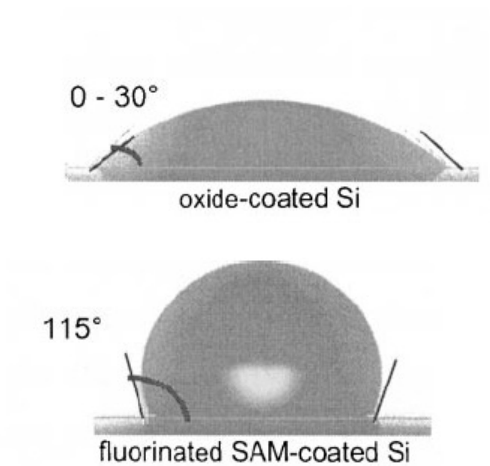
Resonance curves at various oscillator-tip separations:
I=3.3 µm, II=141 nm, III=116 nm, IV=98 nm

Chan et al., "Nonlinear Micromechanical Casimir Oscillator", *Physical reviews letters*, 87, No 21 (2001)

Contact forces: Capillary condensation

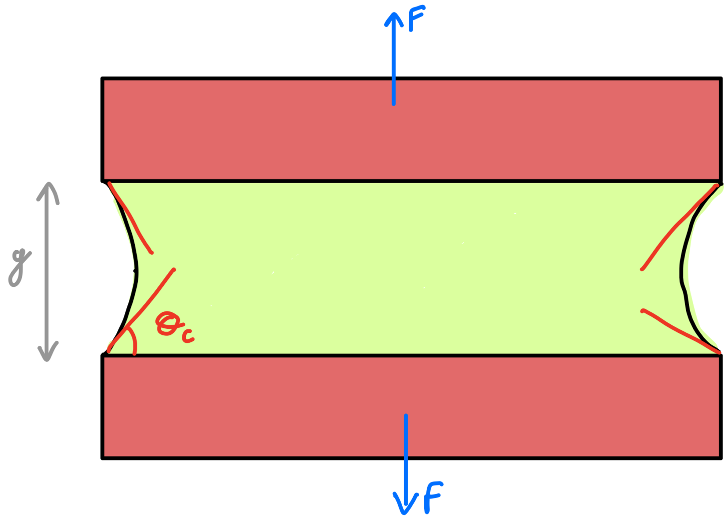
Adhesion due to capillary condensation



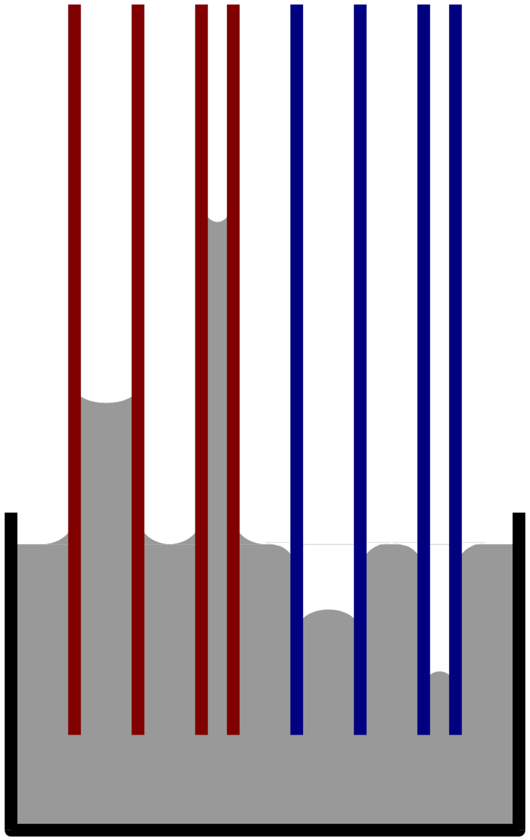


Liquid on a surface either wets or forms a droplet.
Contact angle θ_c given by competing energies of liquid-air, solid-liquid, and solid-air interfaces (surface tensions γ)

$$\gamma_{sa} = \gamma_{sl} + \gamma_{la} \cos(\theta_c)$$



$$F = \frac{2A\gamma_{la} \cos(\theta_c)}{g}$$



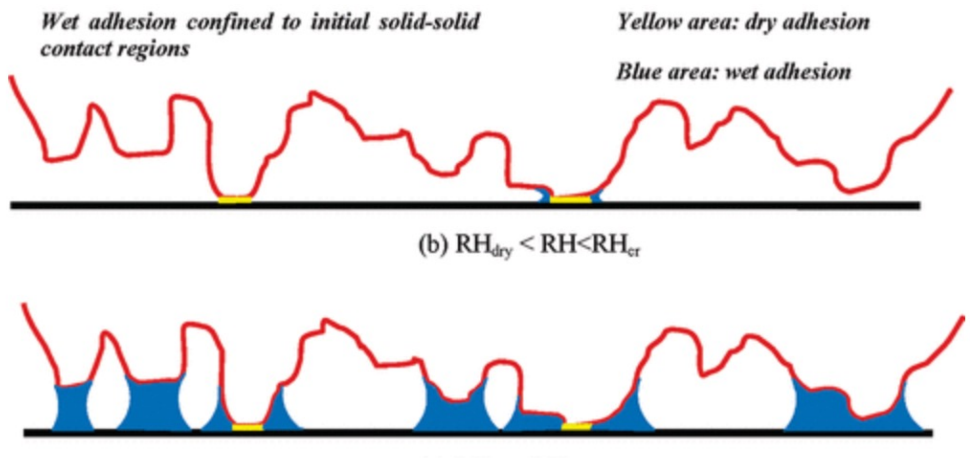
$$\rho g h = \frac{2\gamma\cos\theta}{a}.$$

a: diameter of tube
h: height liquid rises to
γ: Surface tension

https://en.wikipedia.org/wiki/Young%E2%80%93Laplace_equation

Stiction: Capillary condensation

- Energy is of order 50 mJ/m^2 for oxide coated silicon (strong function of roughness, scales with: density of asperities / ln[%RH])
- Lots of modeling work done to include effect of roughness



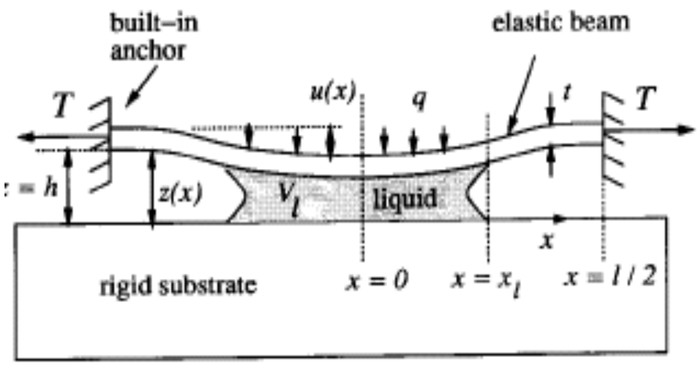
Liquids that wet will spontaneously condense into small cracks, pores, gaps.

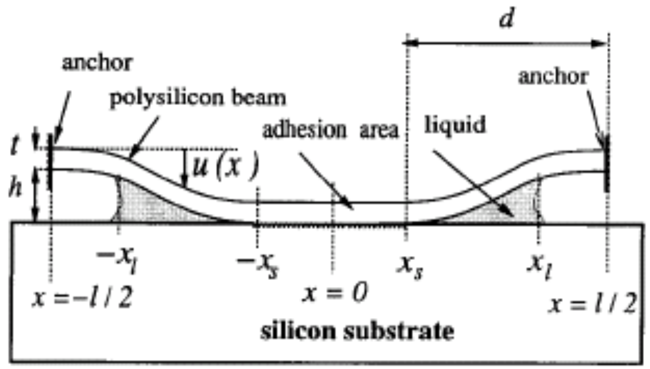
Correct calculation of capillary force depends crucially on detailed geometry on the nm scale (not usually available!)

J. Wang, J. Qian, H. Gao, Effects of Capillary Condensation in Adhesion between Rough Surfaces. *Langmuir* **25**, 11727–11731 (2009).

MEMS Collapse when drying after release

- When the process liquid is evaporating, the surface tension pulls the structure towards the substrate.
- They might then get stuck (for ever…)





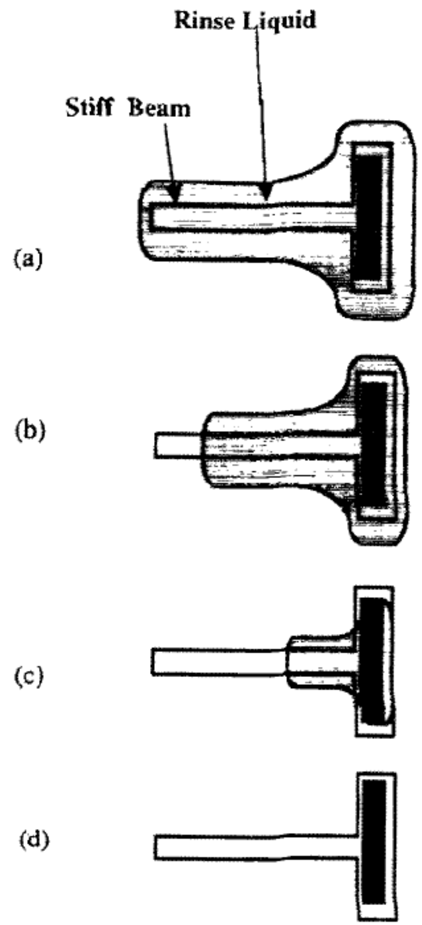


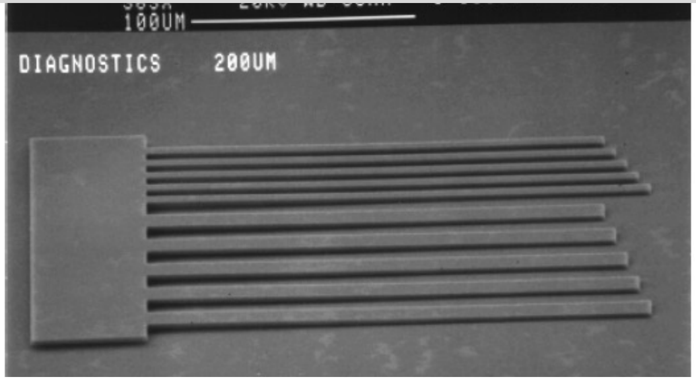
Fig. 7. Evaporation drying of short cantilever.

C. H. Mastrangelo and C. H. Hsu, "Mechanical stability and adhesion of microstructures under capillary forces. II. Experiments," *Journal of Microelectromechanical Systems*, vol. 2, no. 1, pp. 44–55, Mar. 1993, doi: 10.1109/84.232594.

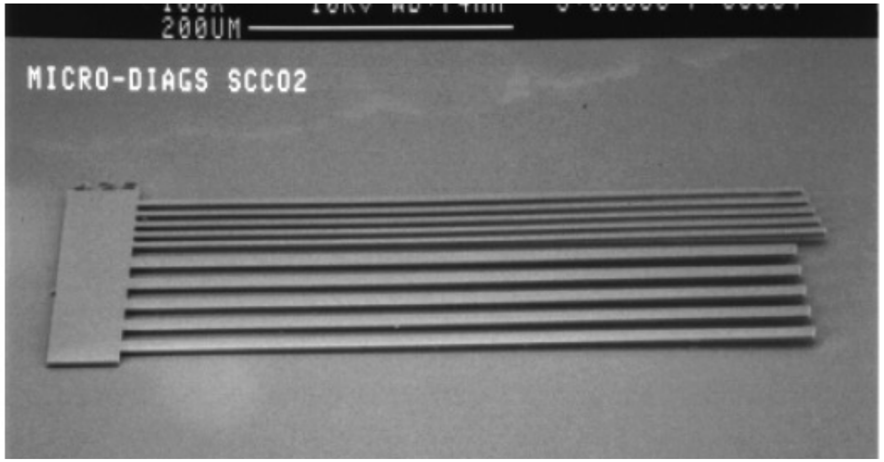
Cantilevers after release, air-dried vs super-critical CO_2 drying

Air-dried cantilever test structures which have all stuck.

The shortest air-dried cantilever is 200 µm long.



$SCCO_2$ extracted cantilevers which are all released and free. The shortest $SCCO_2$ extracted cantilever is 500 µm in length

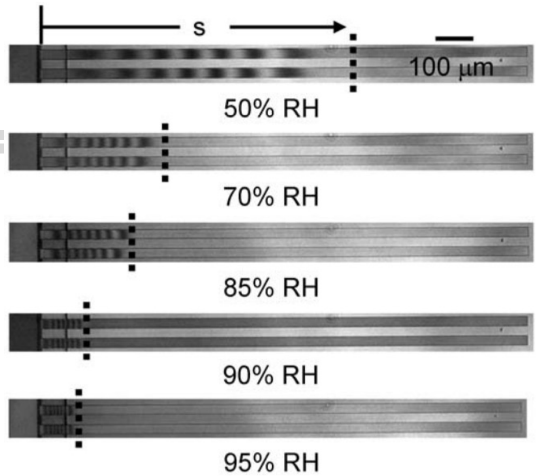


$SCCO_2$ = super critical CO_2 drying
Avoids liquid-gas interface by operating in supercritical region

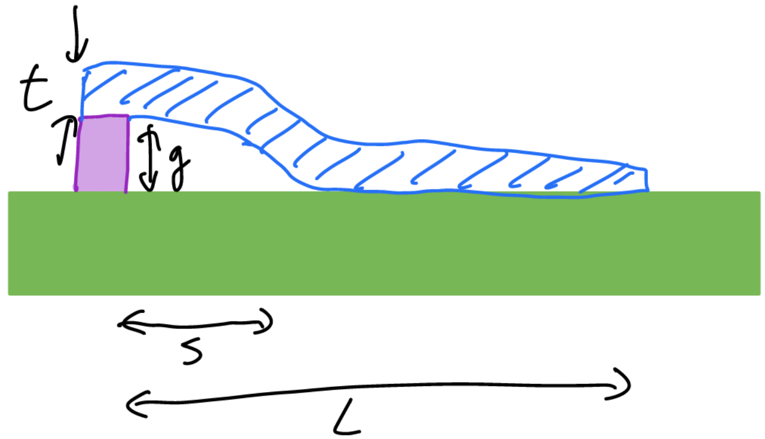
Images courtesy Sandia National Lab

How to measure adhesion energy in MEMS

- Critical length of cantilevers:
  equate *bending energy* with *surface energy*



F. W. DelRio et al, Rough surface adhesion in the presence of capillary condensation. *Applied Physics Letters* **90**, 163104 (2007).



E: Young's modulus
t: thickness
g: initial gap
s: detachment (i.e., non-stuck) length

$$\gamma_{\mathrm{ad}} = \frac{3}{2} E \frac{t^3 g^2}{s^4}$$

To measure work of adhesion per unit surface :

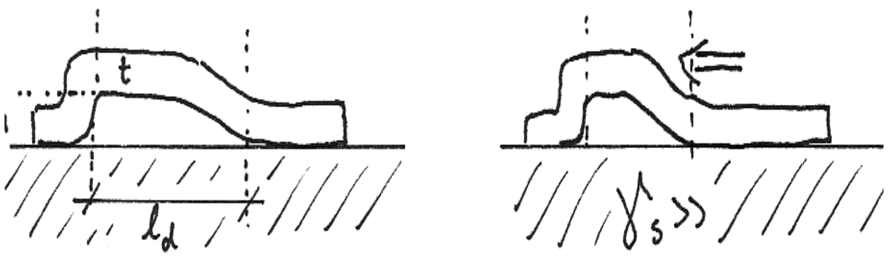
1. Make array of cantilevers of different lengths
2. Force them down, then remove force
3. Use interferometer to measure attachment length and determine surface energy (Caution on accuracy: energy scales as s^4)

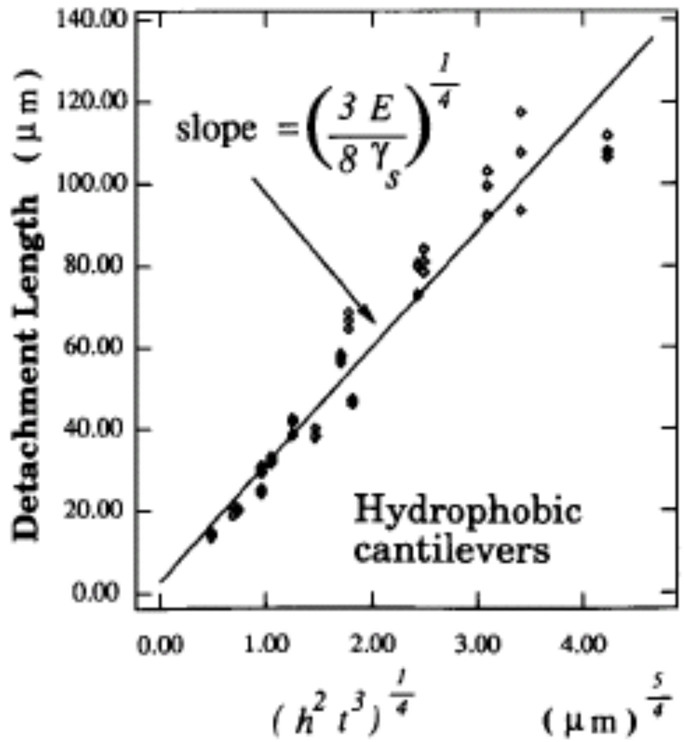
Critical beam length: when surface forces are larger than mechanical restoring forces

Maximum length of beam where can never get stuck

$$L_{max} = \sqrt[4]{\frac{3Eh^3d^2}{8\gamma_s}}$$

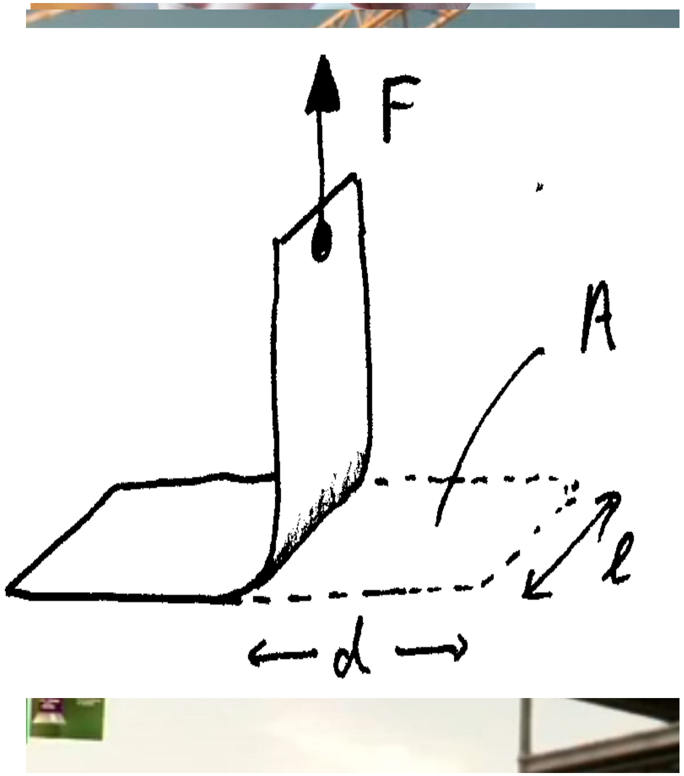
$$L_{max} \propto \left(h^3 \cdot d^2\right)^{1/4} \quad \propto L^{5/4}$$





C. H. Mastrangelo and C. H. Hsu, "Mechanical stability and adhesion of microstructures under capillary forces. II. Experiments," *Journal of Microelectromechanical Systems*, vol. 2, pp. 44–55, 1993, doi: 10.1109/84.232594.

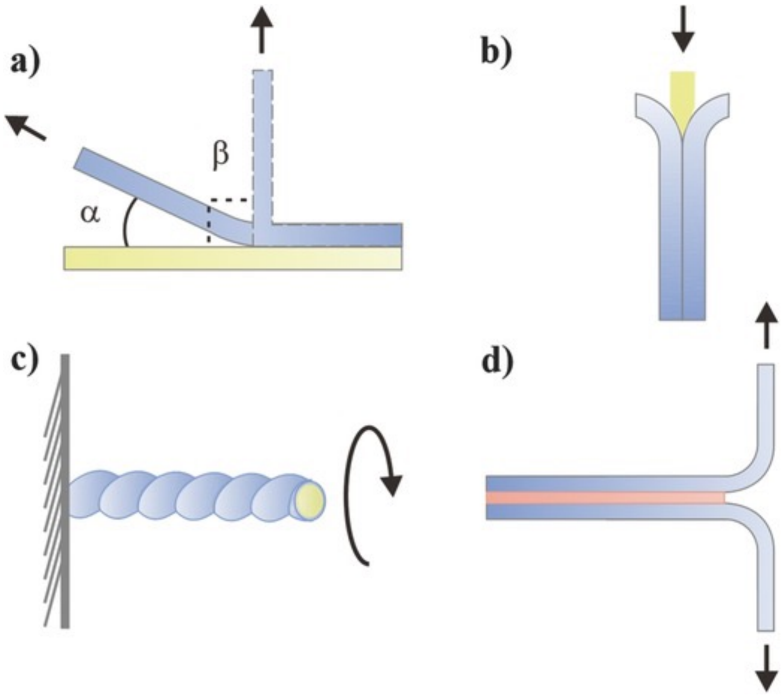
Simple method to measure surface energy for tape



$$E = Fd = \gamma_s A$$

$$\gamma_s = \frac{F_{to.unpeel}}{l}$$

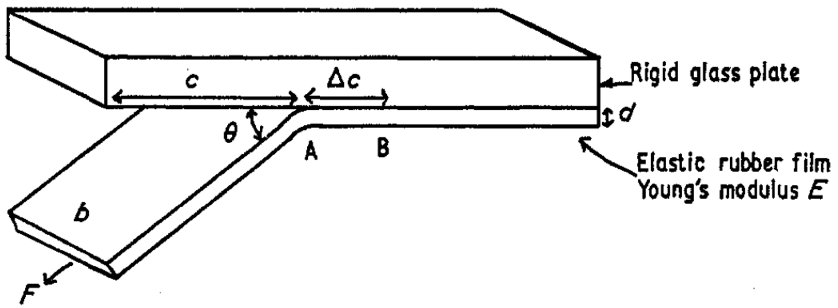
E.g. Scotch tape: l=1 cm, F=0.5 N, so γ_s=50 J/m^2



For testing wafer bonding, the surface energy is measured by inserting a razor blade and measuring the distance of debonding. The parameters are wafer thickness and elastic modulus.

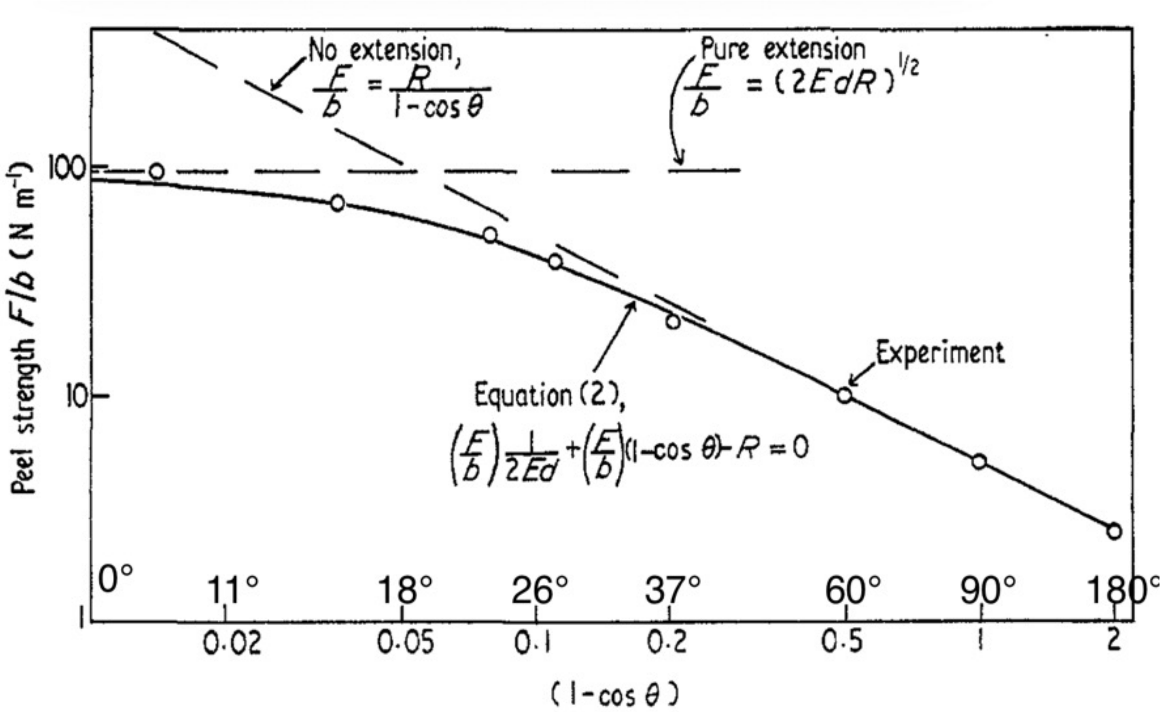
Z. Gu 2016. Adv. Science, vol 3, 1500327
https://onlinelibrary.wiley.com/doi/10.1002/advs.201500327

Peeling mechanics – surface energy and crack propagation well know for adhesive elastic film



R [J/m^2] = surface energy = energy required to create new surface, per unit area.

Crack propagation model



K. Kendall, Thin-film peeling-the elastic term. *J. Phys. D: Appl. Phys.* **8**, 1449–1452 (1975).

How to prevent stiction?

- Avoid collapse (e.g. dry processing)
- Avoid surface charges (e.g. balanced voltage actuation)
- Mechanical design, stiffer (if possible), increase gap (if applicable)
- Decrease contact area (bumps, dimples, roughness)
- Decrease surface energy: films, SAMs: Surface energy is decreased by using teflon-like thin films

MEMSCAP dimples

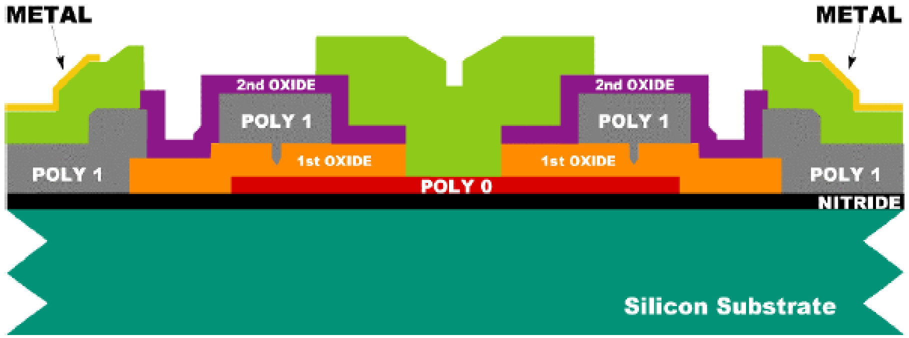


Figure 1.15. The wafer is coated with photoresist and the eighth level (METAL) is lithographically patterned. The metal (gold with a thin adhesion layer) is deposited by lift-off patterning which does not require etching. The side wall of the photoresist is sloped at a reentrant angle, which allows the metal to be deposited on the surfaces of the wafer and the photoresist, but provides breaks in the continuity of the metal over the reentrant photoresist step. The photoresist and unwanted metal (atop the photoresist) are then removed in a solvent bath. The process is now complete and the wafers can be coated with a protective layer of photoresist and diced. The chips are sorted and shipped.

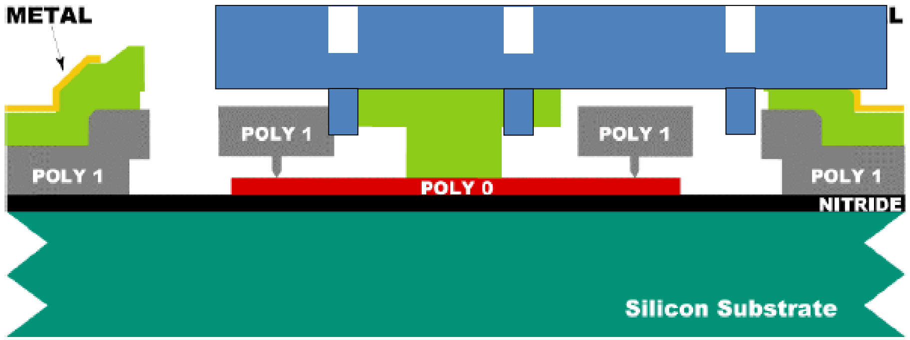
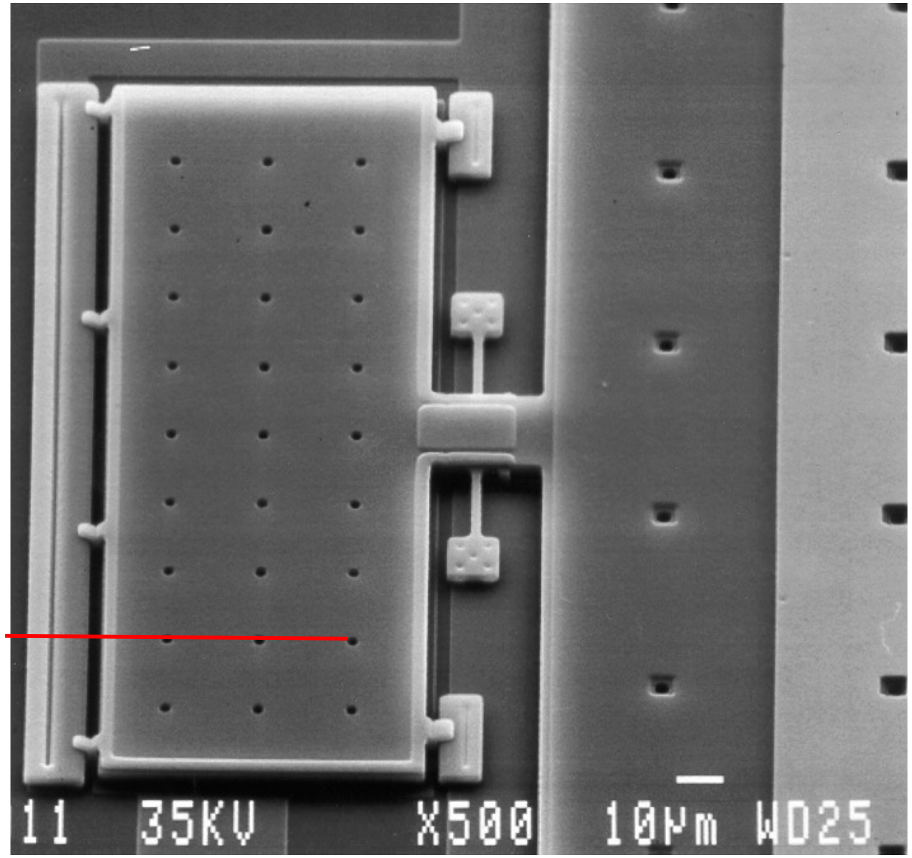
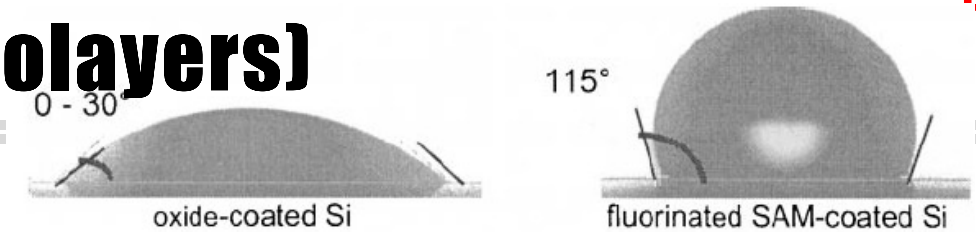
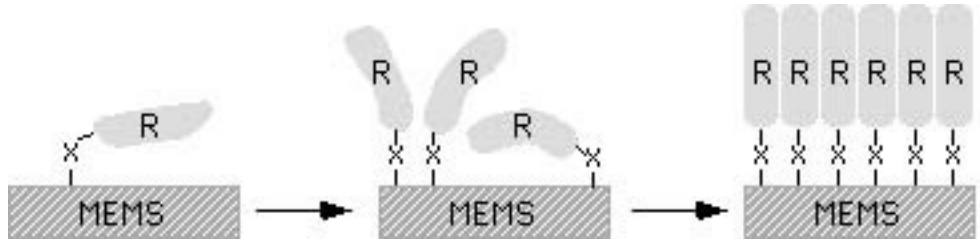


Figure 1.16. The structures are released by immersing the chips in a 49% HF solution. The Poly 1 "rotor" can be seen around the fixed Poly 2 hub. The stacks of Poly 1, Poly 2 and Metal on the sides represent the stators used to drive the motor electrostatically.

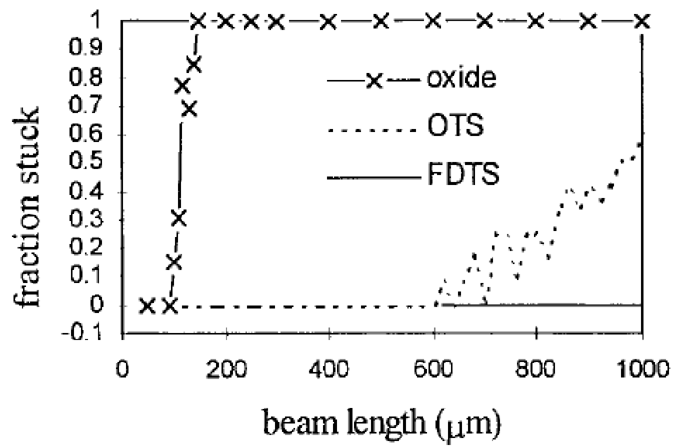


SAM (self-assembled monolayers)





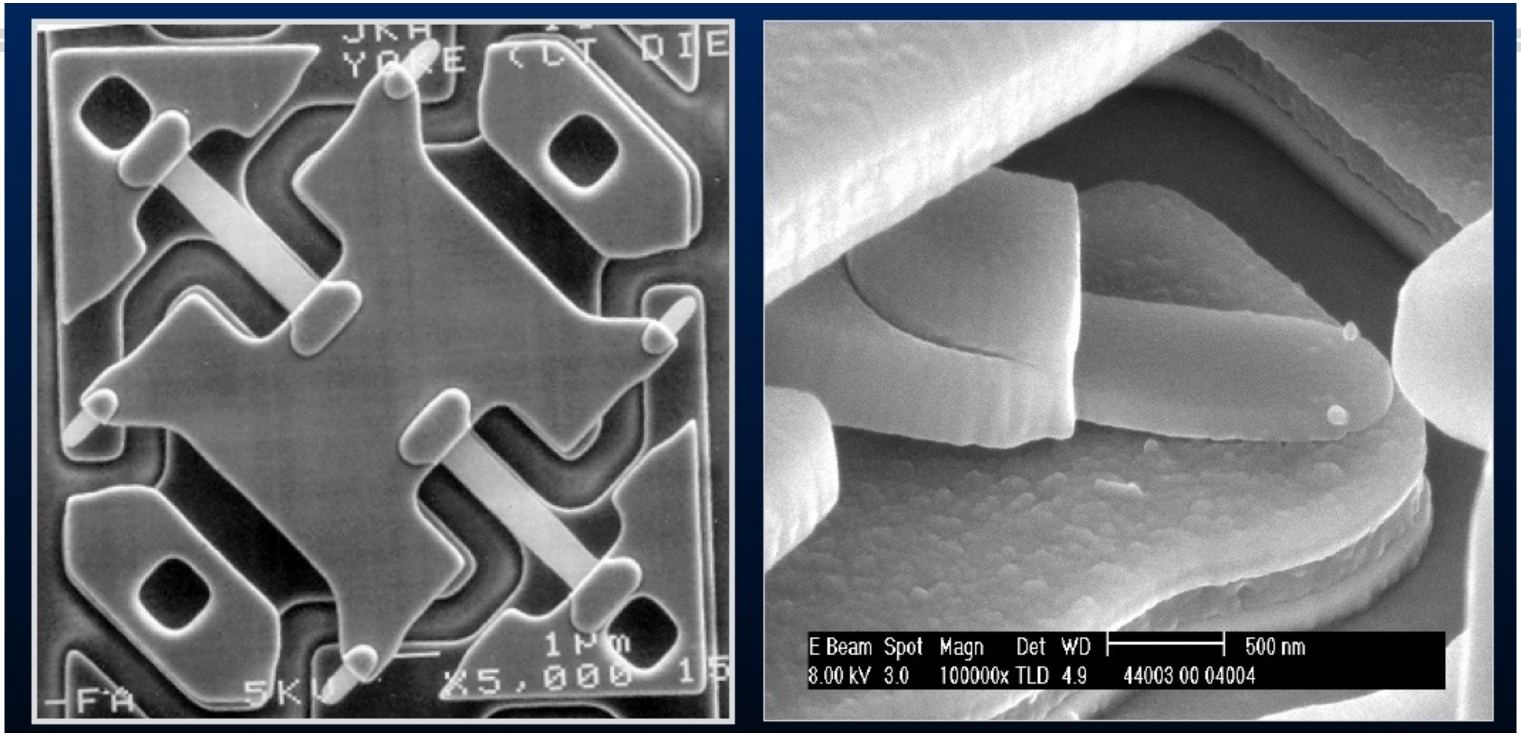
- Can reduce surface tension by several orders of magnitude with surface treatments
- Typically single monolayer of fluorinated molecules
- Energies of 10 μJ/m^2 achieved for Teflon-like coatings vs. 50 mJ/m^2 for untreated



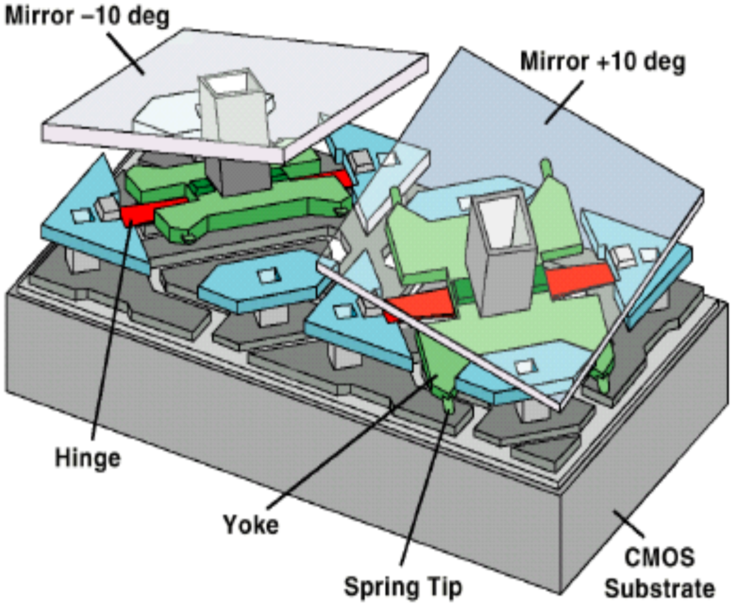
Maboudian R and Howe R, "Critical review: adhesion in surface micromechanical structures", J. Vac. Sci. Technol. B 15 (1997)1–20

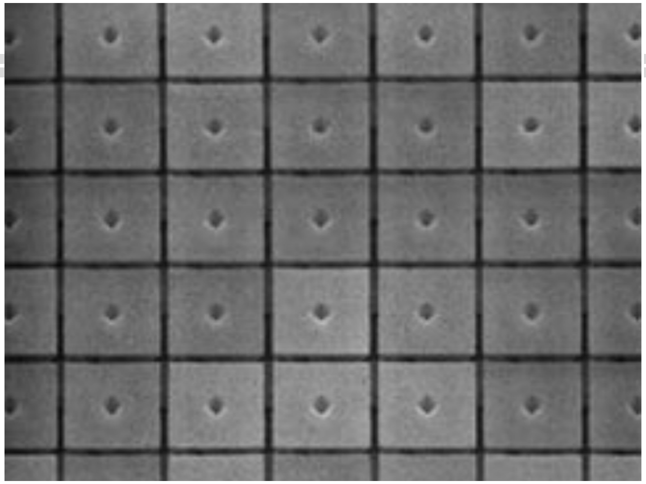
EPFL

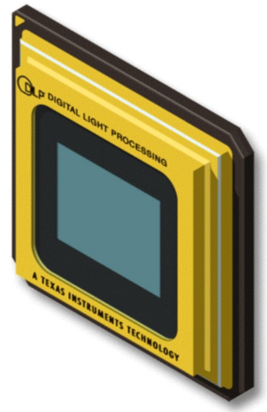
TI DMD mirrors

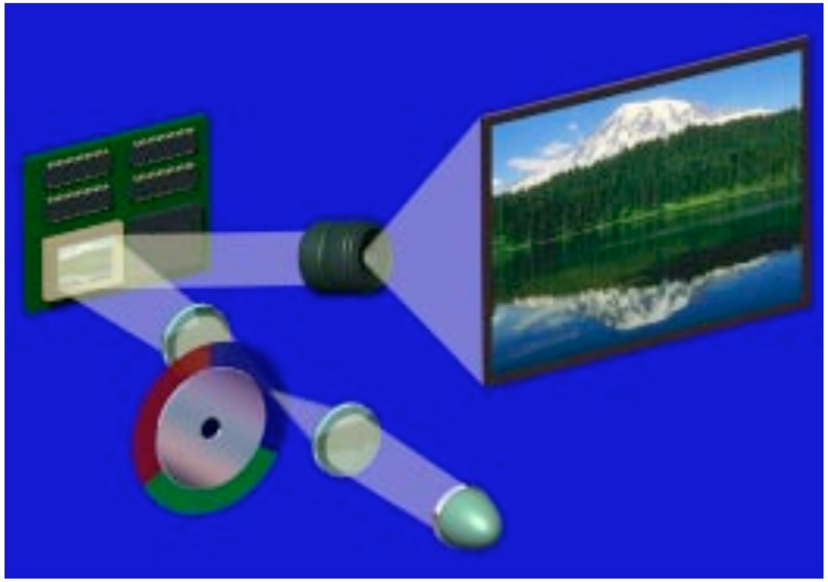


- SAM (self-assembled monolayer) of $CF_3(CF_2)_8COOH$, vapor phase deposited, plus getter strip to maintain desired vapor pressure of SAM source during operation.
- Hermetic package
- Minimized contact area


Mirror –10 deg
Mirror +10 deg
Hinge
Yoke
Spring Tip
CMOS
Substrate




DIGITAL LIGHT PROCESSING
A TEXAS INSTRUMENTS TECHNOLOGY



Cantilever maximum length (due to in-use stiction)

EPFL

- A maximal length on a beam can be set as the length at which the elastic restoring force is sufficient to overcome and surface of gravity forces. Adhesion energy and surface charges are the limiting factors.
- For gravity: $d_0 = \frac{3}{2}\frac{\rho g}{E}\frac{l_{max}^4}{h^2}$ so $l_{max} = \sqrt[4]{\frac{2E}{3\rho g} d_0 h^2} \propto L^{3/4}$

Force	Equation	Maximum length for MUMPs
Acceleration/gravity	$L_{max} = \sqrt[4]{\frac{2Et^2 d}{3g\rho}}$	2.5 mm
Casimir effect	$L_{max} = \sqrt[4]{\frac{24F_c^{*max} Et^3 d^5}{\eta \Re}}$	7.3 mm
Interface charges	$L_{max} = \sqrt[4]{\frac{16\varepsilon_0 Et^3 d}{3\sigma^2}}$	~264 to 835 μm
Substrate voltage	$(VL^2)_{max} = \sqrt{\frac{2Et^3 d^3 F_e^{*max}(\xi)}{\varepsilon_0}}$	1.83 V^{-2} mm^{-1}
Adhesion	$L = \sqrt[4]{\frac{3Et^3 d^2}{8\gamma_s}}$	52 to 907 μm

Force	Scaling
Acceleration/gravity	$L_{max} = l^{3/4}$
Casimir effect	$L_{max} = l^2$
Interface charges	$L_{max} = l$
Substrate voltage	$L_{max} = l^{3/2}$
Adhesion	$L_{max} = l^{5/4}$

R. Johnstone et al., "Theoretical limits on the freestanding length of cantilevers produced by surface micromachining", J. Micromech Microeng 12 (2002) 855

Friction coefficient scaling

- Friction is due to cohesive forces (VdW , …) not directly to roughness.
- Macroscopically, friction forces are proportional to the normal force because this increases the effective “contact area”
- The normal force is sum of: weight + additional proximity forces (e.g. electrostatic)
- **The apparent friction coefficient is larger at small scale because of contact forces <u>add</u> to the normal forces.**

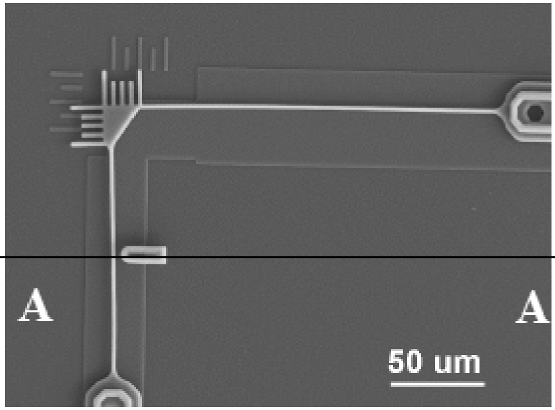
- Normal (dry) friction coefficient definition: $F_F = \mu F_N$
- With surface forces F_{Surf}: $F_F = \mu\left(F_N + F_{surf}\right)$
- Apparent friction coefficient μ^* $\mu^* = \mu \dfrac{F_F + F_{surf}}{F_N}$ (increases at small scale)

Friction coefficient scaling

- At large scale (friction due to own weight): $F_F = \mu Mg \quad \propto L^3$
- At small scale (small F_{Normal}, dominant $F_{Surface}$): $F_F = \mu F_S \quad \propto L^2$
- With lubricants (assuming wet friction) $F_{Ff} \propto \frac{v}{h} \quad \propto L^{-1}$ (at constant speed)

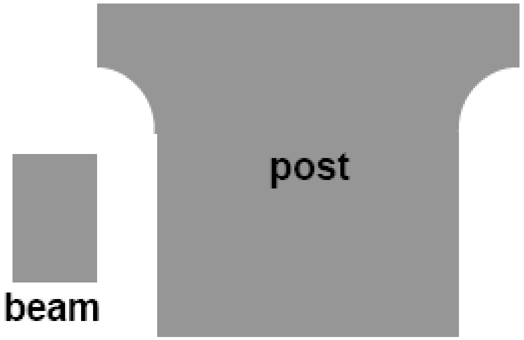
- This model doesn't take in account surface structures (which can dominate for very small contact area and at very small scale)

Sandia National Lab (USA) – MEMS Friction tester

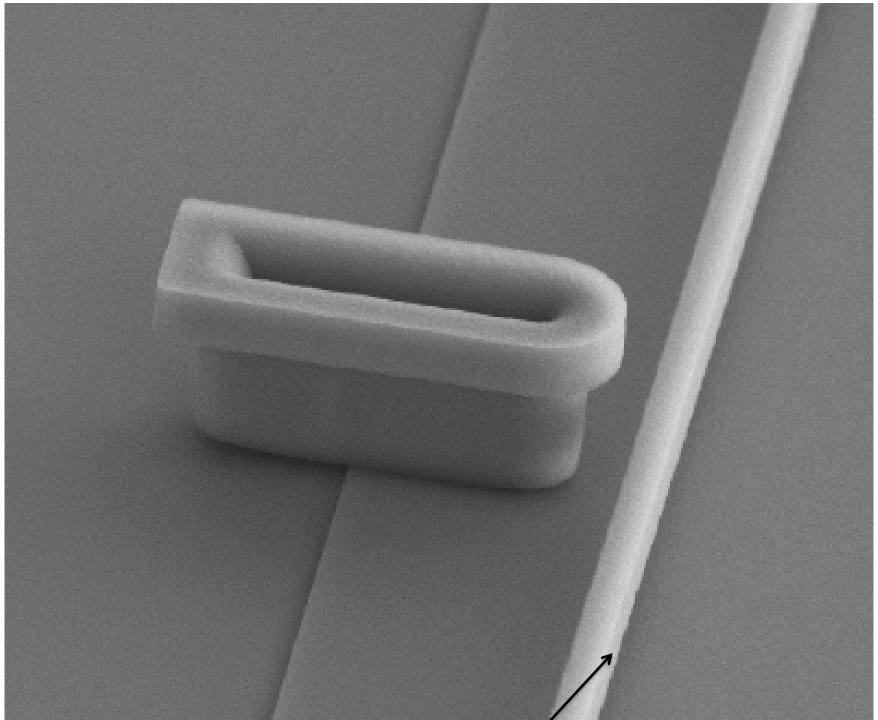


apply load
on post

reciprocate beam
against post



Daan Hein Alsem et al.,
JMEMS Vol. 17, Oct 2008



Poly-Si Beam: 2 μm wide

Control Normal force (~20 μN) , and sideways pulling force